REPRESENTATIONS OF FINITE OSBORN LOOPS

A. O. Isere
Department of Mathematics
Ambrose Alli University, Ekpoma 310001, Nigeria.
J. O. Adéníran & A. A. A. Agboola
Department of Mathematics,
Federal University of Agriculture,
Abeokuta 110101, Nigeria.

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Abstract

It is shown that an Osborn loop of order n has $n/2$ generators. Given the generators such that $R(2)^2 = I$, the representation Π is generated by $R(2) \circ R(2+i) = R(3+i) \forall i = 1, 3, 5, ..., n-3$. The representation of Osborn loops of order 16 is presented and it is used as an example to verify the results. It is also shown that the order of every element of the representation Π divides the order of the loop, hence, Osborn loops of order 16 are langrangelike.
Keywords: Osborn loops, Representation, order, generators, isomorphism

Outline

GROUPOIDS, GROUPS, QUASIGROUPS AND LOOPS

A loop L is a quasigroup with a neutral element. All groups are loops but all loops are not groups. Those that are groups are called associative loops. Thus, loop theory is a generalization of group theory by introducing non-associativity into the set. However, we wish to formally define a loop.

A LOOP

Definition

A loop is a set G with binary operation (denoted here simply by juxtaposition) such that

- *for each a in G, the left multiplication map $L_a : G \to G, x \to ax$ is bijective,*
- *for each a in G, the right multiplication map $R_a : G \to G, x \to xa$ is bijective; and*
- *G has a two-sided identity G.*

The order of G is its cardinality $|G|$.

Definition

A loop $(G, \cdot, /, \backslash, e)$ *is a set* G *together with three binary operations* $(\cdot)$, $(/)$, $(\backslash)$ *and one nullary operation* e *such that*

(i) $x \cdot (x\backslash y) = y$, $(y/x) \cdot x = y$ *for all* $x, y \in G$,

(ii) $x\backslash(x \cdot y) = y$, $(y \cdot x)/x = y$ *for all* $x, y \in G$ *and*

(iii) $x\backslash x = y/y$ *or* $e \cdot x = x$ *and* $x \cdot e = x$ *for all* $x, y \in G$.

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(iii) $x\backslash x=y/y$ *or* $e\cdot x=x$ *and* $x\cdot e=x$ *for all* $x,y\in G$.

Definition

Let G *be a loop. The set* $\Pi=\{R(a):a\in G\}$ *is called the right regular representation of* G *or briefly the representation of* G.

Definition

[2, 3] A set Π of permutations on a set G is the representation of a loop $(G, \cdot)$ if and only if

(i) $I \in \Pi$ *(identity mapping),*

(ii) Π *is transitive on* G *(i.e for all* $x, y \in G$*, there exists a unique* $\pi \in \Pi$ *such that* $x\pi = y$*),*

(iii) *if* $\alpha, \beta \in \Pi$ *and* $\alpha\beta^{-1}$ *fixes one element of* G*, then* $\alpha = \beta$*.*

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The left and right representation of a loop G is denoted by

$$\Pi_\lambda(G,\cdot)=\Pi_\lambda(G) \qquad \text{and} \qquad \Pi_\rho(G,\cdot)=\Pi_\rho(G) \text{ respectively.}$$

Osborn Loops

A loop $I(\cdot)$ is called an Osborn loop if it obeys the identity:

$$(x^{\lambda}\backslash y)\cdot zx = x(yz\cdot x) \qquad (1)$$

for all $x, y, z \in I$. Here x^{λ} is the left inverse of x, and $a\backslash b$ is the left division operation.

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for all $x, y, z \in I$. Here x^{λ} is the left inverse of x, and $a\backslash b$ is the left division operation. The term Osborn loops first appeared in a work of Huthnance Jr [5] in 1968, on generalized Moufang loops. However, the equation (1) above is according to Basarab [5] in 1979[1]. For detail see Kinyon[5] and Jaiyeola[2].

Preliminaries

Theorem

(Huthnance [5] and Basarab) Let G be an Osborn loop. $N_\rho(G) = N_\lambda(G) = N_\mu(G) = N(G)$ *and* $N(G) \trianglelefteq G$.

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An Osborn loop that is flexible or which has the LAP or RAP or LIP or RIP or AAIP is a Moufang loop. But an Osborn loop that is commutative or which has the CIP is a commutative Moufang loop.

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Remark

The theorem helps to determine a non-Moufang Osborn loop.Consider also [2, 3]

Examples of Osborn Loops

Example

(Kinyon [5]) The smallest order for which proper(non-Moufang and non-CC) Osborn loops with non-trivial nucleus exists is 16. *There are two of such loops.*

- *Each of the two is a G-loop.*
- *Each contains as a subgroup, the dihedral group(D_4) of order* 8.
- *For each loop, the center coincides with the nucleus and has order* 2. *The quotient by the center is a non-associative CC-loop of order* 8.

Example (cont'd)

- *The second center is* $\mathbb{Z}_2 \times \mathbb{Z}$, *and the quotient is* $\mathbb{Z}_4$.
- *One loop satisfies* $L_x^4 = R_x^4 = I$, *the other does not.*

Their multiplication tables are presented below in form of acceptable loops as Table 1 and Table 2.

·	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
2	2	1	4	3	6	5	8	7	10	9	12	11	14	13	16	15
3	3	4	1	2	7	8	5	6	11	12	9	10	15	16	13	14
4	4	3	2	1	8	7	6	5	12	11	12	9	16	15	14	13
5	5	6	8	7	1	2	4	3	13	14	16	15	10	9	11	12
6	6	5	7	8	2	1	3	4	14	13	15	16	9	10	12	11
7	7	8	6	5	3	4	2	1	15	16	14	13	12	11	9	10
8	8	7	5	6	4	3	1	2	16	15	13	14	11	12	10	9
9	9	10	11	12	15	16	13	14	5	6	7	8	3	4	1	2
10	10	9	12	11	16	15	14	13	6	5	8	7	4	3	2	1
11	11	12	9	10	13	14	15	16	8	7	6	5	2	1	4	3
12	12	11	10	9	14	13	16	15	7	8	5	6	1	2	3	4
13	13	14	16	15	12	11	9	10	1	2	4	3	7	8	6	5
14	14	13	15	16	11	12	10	9	2	1	3	4	8	7	5	6
15	15	16	14	13	10	9	11	12	4	3	1	2	6	5	7	8
16	16	15	13	14	9	10	12	11	3	4	2	1	5	6	8	7

Table : The first Osborn loop of order 16 that is a G-loop

$\odot$	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
2	2	1	4	3	6	5	8	7	10	9	12	11	14	13	16	15
3	3	4	1	2	7	8	5	6	11	12	9	10	15	16	13	14
4	4	3	2	1	8	7	6	5	12	11	12	9	16	15	14	13
5	5	6	8	7	1	2	4	3	13	14	16	15	10	9	11	12
6	6	5	7	8	2	1	3	4	14	13	15	16	9	10	12	11
7	7	8	6	5	3	4	2	1	15	16	14	13	12	11	9	10
8	8	7	5	6	4	3	1	2	16	15	13	14	11	12	10	9
9	9	10	11	12	15	16	13	14	7	8	5	6	2	1	4	3
10	10	9	12	11	16	15	14	13	8	7	6	5	1	2	3	4
11	11	12	9	10	13	14	15	16	6	5	8	7	3	4	1	2
12	12	11	10	9	14	13	16	15	5	6	7	8	4	3	2	1
13	13	14	16	15	12	11	9	10	3	4	2	1	6	5	7	8
14	14	13	15	16	11	12	10	9	4	3	1	2	5	6	8	7
15	15	16	14	13	10	9	11	12	2	1	3	4	7	8	6	5
16	16	15	13	14	9	10	12	11	1	2	4	3	8	7	5	6

Table : The second Osborn loop of order 16 that is a G-loop

$\cdot/\odot$	1	2	3	4	5	6	7	8
1	1	2	3	4	5	6	7	8
2	2	1	4	3	6	5	8	7
3	3	4	1	2	7	8	5	6
4	4	3	2	1	8	7	6	5
5	5	6	8	7	1	2	4	3
6	6	5	7	8	2	1	3	4
7	7	8	6	5	3	4	2	1
8	8	7	5	6	4	3	1	2

Table : The Smarandache Subgroup(D_4) of an Osborn loop

1	2	3	4	5	6	7	8
2	1	4	3	6	5	8	7
3	4	1	2	7	8	5	6
4	3	2	1	8	7	6	5
5	6	8	7	1	2	4	3
6	5	7	8	2	1	3	4
7	8	6	5	3	4	2	1
8	7	5	6	4	3	1	2

Table : The first latin sub-square of length 8 from the first and second Osborn loops

9	10	11	12	13	14	15	16
10	9	12	11	14	13	16	15
11	12	9	10	15	16	13	14
12	11	10	9	16	15	14	13
13	14	16	15	10	9	11	12
14	13	15	16	9	10	12	11
15	16	14	13	12	11	9	10
16	15	13	14	11	12	10	9

Table : The second latin sub-square of length 8 from the first and second Osborn loops

Construction

Example

(Isere et al[1])
Let $I(\cdot) = C_{2n} \times C_2$ *that is*
$I = \{(x^{\alpha}, y^{\beta}), 0 \leq \alpha \leq 2n-1, 0 \leq \beta \leq 1\}$ *and the binary operation is defined as follows:*

$$(x^a, e) \cdot (x^b, y^{\beta}) = (x^{a+b}, y^{\beta}) \tag{2}$$

$$(x^a, y^{\alpha}) \cdot (x^b, e) = (x^{a+b}, y^{\alpha}) \tag{3}$$

$$(x^a, y^{\alpha}) \cdot (x^b, y^{\beta}) = (x^{a+b}, y^{\alpha+\beta}) \text{ if } b \equiv 0 (mod\ 2) \tag{4}$$

Example

cont'd

$$= (x^{a+3b}, y^{\alpha+\beta}) \text{ if } b \equiv 1(mod\ 2) \quad (5)$$

$$(x^a, y^\alpha) \cdot (x^b, y^\beta) = (x^{a+3b}, y^{\alpha+3\beta}) \text{ if } a \equiv 1(mod\ 2), b \equiv 1(mod\ 2) \quad (6)$$

$$(x^{b+c}, y^\delta) \cdot (x^a, y^\alpha) = (x^{a+b+c}, y^{\alpha+\delta}) \text{ if } b \equiv 0(mod\ 2) \quad (7)$$

$$(x^{b+c}, y^\delta) \cdot (x^a, y^\alpha) = (x^{a+3b+c}, y^{\alpha+\delta}) \text{ if } b \equiv 1(mod\ 2) \quad (8)$$

$$(x^{b+c}, y^{\beta+\gamma}) \cdot (x^a, y^\alpha) = (x^{3a+3b+c}, y^{\alpha+3\beta+\gamma}) \text{ if } a \equiv 1(mod\ 2), b \equiv 1(mod\ 2) \quad (9)$$

Then $I(\cdot)$ *is an Osborn loop of order* $4n$, *where* $n = 2, 3, 4, 6, 9, 12$ *and* 18

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$$(x^a, e) \cdot (x^b, y^{\beta}) = (x^{a+b}, y^{\beta}) \tag{10}$$

$$(x^a, y^{\alpha}) \cdot (x^b, e) = (x^{a+b}, y^{\alpha}) \tag{11}$$

$$(x^a, y^{\alpha}) \cdot (x^b, y^{\beta}) = (x^{a+b}, y^{\alpha+\beta}) \text{ if } b \equiv 0 (mod\ 2) \tag{12}$$

Example

cont'd

$$= (x^{a+kb}, y^{\alpha+\beta}) \text{ if } a \equiv 0(mod\ 2), b \equiv 1(mod\ 2) \tag{13}$$

$$(x^a, y^\alpha)\cdot(x^b, y^\beta) = (x^{a+kb}, y^{\alpha+k\beta}) \text{ if } a \equiv 1(mod\ 2), b \equiv 1(mod\ 2) \tag{14}$$

$$(x^{b+c}, y^\delta)\cdot(x^a, y^\alpha) = (x^{a+b+c}, y^{\alpha+\delta}) \text{ if } a \equiv 0(mod\ 2), b \equiv 0(mod\ 2) \tag{15}$$

$$(x^{b+c}, y^\delta)\cdot(x^a, y^\alpha) = (x^{a+kb+c}, y^{\alpha+\delta}) \text{ if } a \equiv 0(mod\ 2), b \equiv 1(mod\ 2) \tag{16}$$

$$(x^{b+c}, y^{\beta+\gamma})\cdot(x^a, y^\alpha) = (x^{b+c+ka}, y^{\beta+\gamma+k\alpha}) \text{ if } a \equiv 1(mod\ 2), b \equiv 0(mod \tag{17}$$

$$(x^{b+c}, y^{\beta+\gamma})\cdot(x^a, y^\alpha) = (x^{c+ka+kb}, y^{\alpha+k\beta+\gamma}) \text{ if } a \equiv 1(mod\ 2), b \equiv 1(mo \tag{18}$$

Then $I(\cdot)$ is an Osborn loop of order $4n$, where

Main Results

Theorem

Let Π *be the right regular representation of an Osborn loop of order* 16*. Then, the element* $R(2)$ *and other elements of odd numbers greater than* 2 *(*$R(3)$,$R(5)$,..,$R(15)$*) that are between* 1 *and* 16 *generate the loop.*

Proof:

Consider an Osborn loop of order 16 represented by Π. Suppose $R(2) \in \Pi$ is given and suppose other elements of odd numbers greater than 2 that are between 1 and 16 (R(3),R(5),R(7),R(9), R(11), R(13) and R(15)) are also given.

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$R(2)^2 = R(3)^2 = R(1) = I$

$$R(4) = R(2) \circ R(3)$$

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$$R(12) = R(2) \circ R(11)$$

$$R(14) = R(2) \circ R(13)$$

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Corollary

Let Π *be the representation of an Osborn loop of order* 16, *and a transposition permutation* $R(2) \in \Pi$ *such that* $R(2)^2 = I$. *Then, given the generators in* Π,

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$R(2) \circ R(2+i) = R(3+i)\ \forall\ i = 1, 3, 5, ..., 13.$, *and* $R(2+i)$ *determines the structure and order of* $R(3+i)$.

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Corollary

Let Π be the representation of an Osborn loop of order 16, and a transposition permutation $R(2) \in \Pi$ such that $R(2)^2 = I$. Then, given the generators in Π, others are generated by: $R(2) \circ R(2+i) = R(3+i)\ \forall\ i = 1, 3, 5, ..., 13.$, and $R(2+i)$ determines the structure and order of $R(3+i)$. i.e. $R(3+i)$ retains the structure and order of $R(2+i)$ where $i = 1, 3, 5, .., 13.$

Remark

If the given generators are of even numbers then the equation becomes $R(2) \circ R(2+i) = R(1+i)\ \forall\ i = 0, 2, 4, ..., 14.$

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$$R(2) \circ R(7) = R(8)$$

continuing in this way up to $i = 13$, we have:

$$R(2) \circ R(15) = R(16)$$

The composition follows from the theorem above. Hence, the proof.

Example

Given the following:

$$R(2) = (1,2)(3,4)(5,6)(7,8)(9,10)(11,12)(13,14)(15,16)$$

$$R(3) = (1,3)(2,4)(5,8)(6,7)(9,11)(10,12)(13,16)(14,15)$$

$$R(5) = (1,5)(2,6)(3,7)(4,8)(9,15,10,16)(11,13,12,14)$$

$$R(7) = (1,7,2,8)(3,5,4,6)(9,13)(10,14)(11,15)(12,16)$$

$$R(9) = (1,9,7,15,2,10,8,16)(3,11,6,14,4,12,5,13)$$

Example

cont'd

$$R(11) = (1, 11, 8, 13, 2, 12, 7, 14)(3, 9, 5, 16, 4, 10, 6, 15)$$

$$R(13) = (1, 13, 6, 9, 2, 14, 5, 10)(3, 15, 7, 12, 4, 16, 8, 11)$$

$$R(15) = (1, 15, 6, 12, 2, 16, 5, 11)(3, 13, 7, 9, 4, 14, 8, 10)$$

determine an Osborn loop of order 16

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$$R(8) = (1,8,2,7)(3,6,4,5)(9,14)(10,13)(11,16)(12,15)$$

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$$R(8) = (1,8,2,7)(3,6,4,5)(9,14)(10,13)(11,16)(12,15)$$

$$R(10) = (1,10,7,16,2,9,8,15)(3,12,6,13,4,11,5,14)$$

Solution

$$R(12) = (1, 12, 8, 14, 2, 11, 7, 13)(3, 10, 5, 15, 4, 9, 6, 16)$$

Solution

$$R(12) = (1, 12, 8, 14, 2, 11, 7, 13)(3, 10, 5, 15, 4, 9, 6, 16)$$

$$R(14) = (1, 14, 6, 10, 2, 13, 5, 9)(3, 16, 7, 11, 4, 15, 8, 12)$$

$$R(16) = (1, 16, 6, 11, 2, 15, 5, 12)(3, 14, 7, 10, 4, 13, 8, 9)$$

Thus, $R(1), ..., R(16)$ *is an Osborn loop of order* 16

Corollary

Let Π be the representation of an Osborn loop of order 16, *and* $R(2)$ *a transposition permutation in* Π. *Given the generators in* Π, *others are generated by* $< R(2), R(2+i) >$ *where* i *is either an even or odd number depending on whether the given generators are of either even or odd number.*

Proof:
Obviously, as $i = 1, 3, 5, ..., 13$, the generators are given, and the proof follows from the theorem and corollary above.

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Remark

For Osborn loops of order n, given the generators, others will be generated by $< R(2), R(2+i) > \ \forall\ i = 1, 3, ..., n-3$ *or* $i = 0, 2, ..., n-2$ *depending on the given generators.*

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Corollary

If Π *is the representation of an Osborn loop of order* n, Π *has* $n/2$ *generators.*

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Given $R(2)$ in Π, the odd numbers between 1 and n that are greater than 2 will be $(n-2)/2$ (i.e. n less 1 and 2).

Corollary

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Given $R(2)$ in Π, the odd numbers between 1 and n that are greater than 2 will be $(n-2)/2$ (i.e. n less 1 and 2).Then adding that of $R(2)$ to this number gives $(n-2)/2+1=n/2$ implies $1/2(n)$ generators.

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We need to show by induction that it is true for all values of n.

Suppose $n = 16$, then by the theorem above, there are 8 generators, which implies $1/2(16) = 16/2$. So it is true for $n = 16$.

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Suppose $n = k + 1$, then, we have $(k + 1)/2 = k/2 + 1/2 = 1/2(k + 1)$. So, it is true for $n = k + 1$.

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Suppose $n = k + 1$, then, we have $(k + 1)/2 = k/2 + 1/2 = 1/2(k + 1)$. So, it is true for $n = k + 1$. Inductively, it is true for all values of n. The proof is complete.

Theorem

Let Π *be the representation of an Osborn loop of order* 16*. Every permutation in* Π *has no distinct inverse.*

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Considering the Osborn loops generated in the example above(the only two examples at that order). We observe that they have no distinct inverses.

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The representation of a finite Osborn loop do not generate a multiplicative gruop.

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The representation of a finite Osborn loop do not generate a multiplicative gruop.

The proof follows from the above theorem

Remark

The above Corollary is confirmed by LOOPs Package in GAP [4]

Lemma

Let Π be the representation of an Osborn loop of order 16*. The order of every element of the representation Π divides the order of the loop.*

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Let Π *be the representation of an Osborn loop of order* 16. *The order of every element of the representation* Π *divides the order of the loop.*

Proof:

The order of elements of the first example of Osborn loop of order 16 are 2 and 4 while the order of elements of the second example are 2, 4 and 8 respectively. These are divisors of 16. The proof follows.

Summary

This work divides the search space of an Osborn loop by 2. One only need to generate the generators by any means and using the equation in the corollary above one can get the entire loop.

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A.M. Asiru (2008),*A study of the classification of finite Bol loops*, Ph.D thesis university of Agriculture, Abeokuta.

A. S. Basarab (1967), *A class of WIP-loops*, Mat. Issled. 2(2), 3-24.

A. S. Basarab(1968), *A certain class of $\mathcal{G}$-loops*, Mat. Issled. 3, 2, 72–77.

A. S. Basarab (1973), *The Osborn loop*, Studies in the theory of quasigroups and loops, 193. Shtiintsa, Kishinev, 12–18.

A. S. Basarab and A.I. Belioglo (1979), *UAI Osborn loops*, Quasigroups and loops, Mat. Issled. 51, 8–16.

V. O. Chiboka and A. R. T. Solarin (1991), *Holomorphs of conjugacy closed loops*, Scientific Annals of Al.I.Cuza. Univ. 37, 3, 277–284.

F. Fenyves (1968), *Extra loops I*, Publ. Math. Debrecen, 15, 235–238.

F. Fenyves (1969), *Extra loops II*, Publ. Math. Debrecen, 16, 187–192.

GAP-Groups Algorithms and Programmes. R. Naggy and Vojtechovsky.

E. D. Huthnance Jr.(1968), *A theory of generalised Moufang loops*, Ph.D. thesis, Georgia Institute of Technology.

A. O. Isere, J. O. Adeniran and A. R. T. Solarin (2012),*Somes Examples Of Finite Osborn Loops*, Journal of Nigerian Mathematical Society, Vol. 31, 91-106.

T.G. Jaiyeola (2008), *The study of the universality of Osborn loops*, Ph.D. thesis, University of Agriculture, Abeokuta

T.G. Jaiyeola And J.O. Adeniran (2009), *New identities in universal Osborn loops, Quasigroups and Related Systems*, Moldova 17(1)

T.G. Jaiyeola And J.O. Adeniran (2009), *Not Every Osborn loop is Universal*, Acta Math. Acad. Paed. Nviregvhaziensis (Hungary)-25, 189-190.

M. K. Kinyon (2005), *A survey of Osborn loops*, Milehigh conference on loops, quasigroups and non-associative systems, University of Denver, Denver, Colorado.

- M. K. Kinyon and J. D. Phillips (2005), *Rectangular loops and rectangular quasigroups*, Comput. Math. Appl. 49, 1679–1685.
- K. Kunen (2000), *The structure of conjugacy closed loops*, Trans. Amer. Math. Soc. 352, 2889–2911.
- LOOPS: Computing with quasigroups and loops in GAP version 1.5.0, package for for GAP 4 by Gabor Naggy and Petr Vojtechovsky April 6 2007. available at http://www.math.du.edu/loops.
- B.D. McKay, A. Meynert and W.Myrvold, *Counting Small Latin Squares*, available at http:// www.cs.uvic.ca

J. M. Osborn (1961), *Loops with the weak inverse property*, Pac. J. Math. 10, 295–304.

H. O. Pflugfelder (1990), *Quasigroups and loops : Introduction*, Sigma series in Pure Math. 7, Heldermann Verlag, Berlin, 147pp.

J. D. Phillips (2006), *A short basis for the variety of WIP PACC-loops*, Quasigroups and Related Systems 1, 14, 73–80.

J. D. Phillips and P. Vojtěchovský (2005), *The varieties of loops of Bol-Moufang type*, Alg. Univer. 3(54), 259–383.