3rd Mile High Conference on Nonassociative Mathematics
Problem Session
August 16, 2013

In order of appearance on stage:

Michael Kinyon:
Conjecture: Let Q be a loop with $\mathrm{Inn}(Q)$ abelian. Then:
(i) $Q/N(Q)$ is an abelian group.
(ii) $Q/Z(Q)$ is a group.

Jonathan Smith:
A finite simple quasigroup is *properly simple* if its multiplication group does not act doubly transitively.

Meta-problem: Replace "simple" with "properly simple" in classifications of classes of simple quasigroups.

Aleš Drápal:
Loop theory cannot be regarded as mature if free objects remain totally obscure. Characterize free Moufang loops, Bol loops, CC loops, LCC loops.

Pavel Kolesnikov:
The following result holds in groups, Lie algebras and associative algebras.

Freiheitssatz: For a variety V and set of generators X, let $V\langle X\rangle$ be the free algebra in V generated by X. Suppose that $f \in V\langle t, x_1, \cdots, x_n\rangle$ and $f \notin V\langle x_1, \ldots, x_n\rangle$. Then $(f) \cap V\langle x_1, \ldots, x_n\rangle = \{0\}$.

Does the result hold in Zinbiel algebras?

Fedir Sokhatsky:
Let A be a class of quasigroups. Then $\sigma \in S_3$ is a *symmetry* of A if A is closed under σ-parastrophes.

The set of all symmetries of A is a subgroup of S_3. A class A is called *skew-symmetric* if its symmetries form the group A_3.

Problem: Find a skew-symmetric variety or prove that it does not exist.

Alberto Elduque:
Let $\mathcal{L}$ be a Lie algebra. Let $\mathcal{L} = \bigoplus_{s\in S} \mathcal{L}_s$ be a grading such that for all s_1, $s_2 \in S$ there is $s_3 \in S$ such that $[\mathcal{L}_{s_1}, \mathcal{L}_{s_2}] \subseteq \mathcal{L}_{s_3}$. This defines a partial binary operation $S \times S \to S$, $(s_1, s_2) \mapsto s_1 * s_2 = s_3$ if $0 \neq [\mathcal{L}_{s_1}, \mathcal{L}_{s_2}] \subseteq \mathcal{L}_{s_3}$.

For a while it was thought that there exists a semigroup G and a one-to-one mapping $f : G \to G$ such that $f(s_1 * s_2) = f(s_1)f(s_2)$. There are counterexamples. Is the statement true for simple Lie algebras?

Jonathan Smith:
Let Q be a Moufang loop of invertible real octonions. Which variety of Moufang loops is generated by Q? All of Moufang loops?

Peter Plaumann:
Does Schreier's inequality hold for loops? That is, does there exist a function $\beta(d, n)$ such that $\mathrm{rank}(R) \leq \beta(d, n)$ whenever Q is a finitely generated loop, $R \leq Q$, $\mathrm{rank}(Q) = d < \infty$ and $[Q : R] = n < \infty$?

Notes: β might depend on the variety. It is true for groups.

Tony Sudbery:
Consider the magic square over $\mathbb{R}$ with split algebras. Why do 4×4 matrices appear in the doubly split magic square as the 3×3 bottom right corner? Is there a conceptual reason?

David Stanovský:
Let Q be a loop.

1) If Q is finite and congruence solvable, does it follow that $\mathrm{Inn}(Q)$ is solvable?
2) If $\mathrm{Mlt}(Q)$ is congruence solvable, does is follow that Q is solvable?
3) If $\mathrm{Inn}(Q)$ is nilpotent, does it follow that Q is nilpotent?
4) If $\mathrm{Inn}(Q)$ is abelian, does it follow that Q is nilpotent?

Gábor Nagy:
We can associate a transversal design with a Latin square: there are 3 classes of points, blocks have size 3, for every 3 points in different classes there is a block containing them. We say that a transversal design has a *projective realization* if it is a subset of some $PG(2,\mathbb{C})$.

Which Latin squares have projective realizations? (This question makes sense up to isotopy.)

Notes: The nonassociative loops of order 5 have projective realizations. For order 6, some do, some don't. For groups the problem is solved: the group must be abelian or dihedral or Q_8.