2-tangle replacements and adequate diagrams

Jung Hoon Lee
(joint with W. S. Kim and S. Lee)
Chonbuk National University

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Outline

1. **Bracket polynomial**

2. **Adequate diagrams**

3. **Adequate 2-tangle diagrams**

4. 2-**tangle replacements**

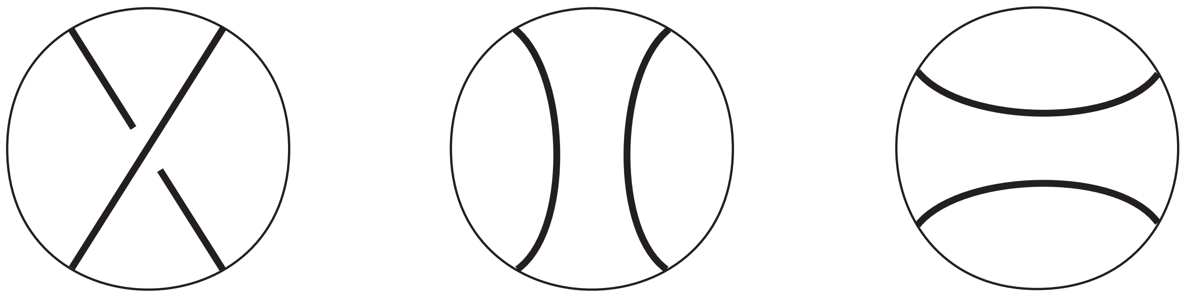
5. **Main results**

• Bracket polynomial

D : an unoriented link (or knot) diagram

The *bracket polynomial* $\langle D \rangle$ is a Laurent polynomial in a variable A defined by the following rules.

1. $\langle \bigcirc \rangle = 1$, where $\bigcirc$ denotes the unknot with no crossings.
2. $\langle D \sqcup \bigcirc \rangle = \delta \langle D \rangle$, where $\delta = -A^{-2} - A^{2}$.
3. $\langle D \rangle = A \langle D_\infty \rangle + A^{-1} \langle D_0 \rangle$.



D $\quad$ D_∞ $\quad$ D_0

If D is an oriented diagram of a link L and $|D|$ is D with its orientation ignored, then the normalized bracket polynomial

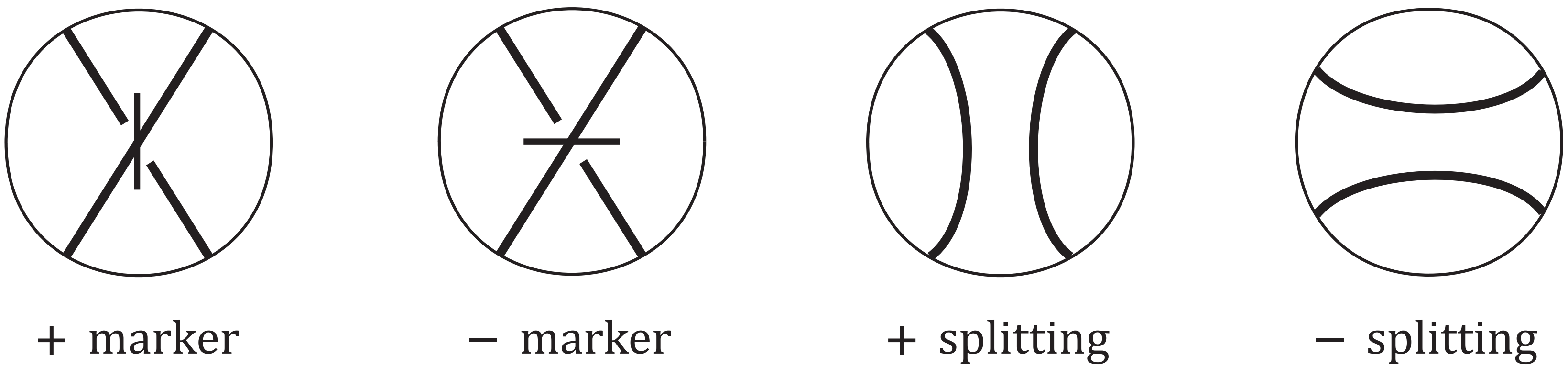
$$V_L(A) = (-A^{-3})^{w(D)} \langle |D| \rangle$$

is a link invariant.
(Here $w(D)$ is the writhe of D.)

When $A = t^{-\frac{1}{4}}$, $V_L(t)$ is the *Jones polynomial*.

A choice of marker for every crossing of an unoriented diagram D is called a *state*.
The result of a splitting is a disjoint union of *state circles*.
$|s|$: the number of state circles for a state s
$p(s)$: the number of $+$ markers in s
$n(s)$: the number of $-$ markers in s

$$\langle D \rangle = \sum_s A^{p(s)}(A^{-1})^{n(s)}\delta^{|s|-1}$$



• Adequate diagrams

D : an unoriented link diagram

s_+ : the state of D in which all markers are $+$
s_- : the state of D in which all markers are $-$

D is $+$*adequate* if, at each crossing, the two strands of the $+$ splitting of s_+ belong to different state circles.

Similarly, $-$*adequate* is defined. ($-$ splitting of s_-)

D is *adequate* if it is both $+$adequate and $-$adequate.

- A reduced alternating diagram is adequate.

- An r-fold parallel of an adequate diagram is adequate.

- $\text{maxdeg}\langle D\rangle = c(D) + 2|s_+| - 2.$

- $\text{mindeg}\langle D\rangle = -c(D) - 2|s_-| + 2.$

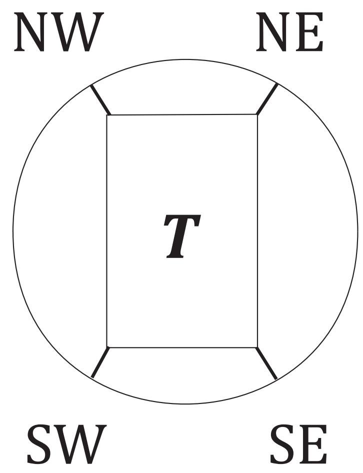
- An adequate diagram has minimal crossing number.

($\exists$ non-adequate minimal crossing diagrams,
e.g. some pretzel links.)

• 2-tangle diagrams

T : an unoriented 2-tangle diagram

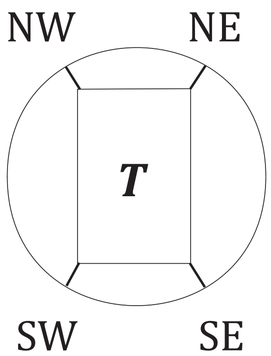
s_+ : the state of T in which all markers are $+$
s_- : the state of T in which all markers are $-$



• Adequate 2-tangle diagrams

T is *+adequate* if the following holds.

1. The splitting of s_+ connects NW to SW and NE to SE.
2. At each crossing of T, the two strands of the + splitting of s_+ belong to different state circle or arc components.

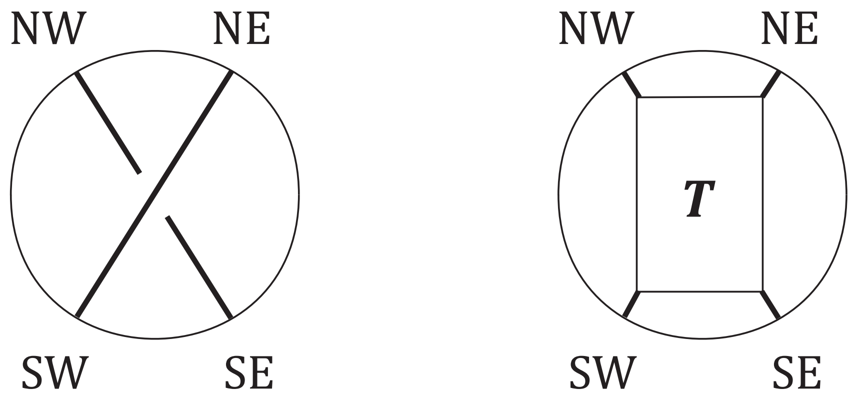


Similarly, *−adequate* is defined. (NW to NE and SW to SE)

T is *adequate* if it is both +adequate and −adequate.

• 2-tangle replacements

We replace a chosen crossing of D by T so that the labels match, and it is called a *2-tangle replacement*, denoted by D_T.



Motivation: Y. Bae considered a link diagram $D \otimes T$ obtained by replacing every crossing of D by T, called a *link diagram with local symmetry*.

- **Main results**

Theorem
If a link diagram D and a 2-tangle diagram T are adequate, then the diagram D_T of a 2-tangle replacement is also adequate.

This gives a way to obtain infinitely many new minimal crossing diagrams.

Sketch of proof)

We show that D_T is $+$adequate. ($-$adequate is similar.)

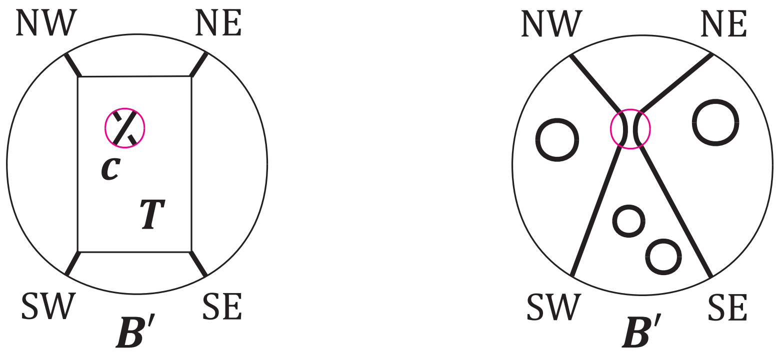
Let B' be a 3-ball defining the tangle T.

Consider a crossing c of D_T.

Case 1. c **is in** T**.**

Case 1.1. Both strands of the $+$ splitting at c belong to state circle components in B'. ($\because T$ is $+$adequate.)

Case 1.2. Both strands of the $+$ splitting at c belong to properly embedded arc components in B'.



Case 1.3. One strand of the $+$ splitting at c belongs to a state circle in B' and the other belongs to a properly embedded arc in B'.

Case 2. c is not in T.

Case 2.1. Both strands of the $+$ splitting at c belong to state circle components in the complement of B'.

Case 2.2. Both strands of the $+$ splitting at c belong to properly embedded arc components in the complement of B'.

Case 2.3. One strand of the $+$ splitting at c belongs to a state circle in the complement of B' and the other belongs to a properly embedded arc in the complement of B'.

Thank you for your attention.