

In-class, Week 5, day 2

Section 5.2, Problem 4: Let $P(n)$ be the statement that a postage of n cents can be formed using just 4-cent stamps and 10-cent stamps. The parts of this exercise outline a strong induction proof that $P(n)$ is true for $n \geq 18$

- a. Show statements $P(18)$, $P(19)$, $P(20)$, $P(21)$ are true, completing the basis step of the proof.
- b. What is the inductive hypothesis?
- c. What do you need to prove in the inductive step?
- d. What do you need to prove in the inductive step?
- e. Complete the inductive step for $k \geq 21$.
- f. Explain why these steps show that this inequality is true whenever $n \geq 18$.

Section 5.2, Problem 14: Suppose you begin with a pile of n stones and split this into n piles of one stone each by successively splitting a pile of stones into two smaller piles. Each time you split a pile, you multiply the number of stones in each of the smaller piles you form, so that if these piles have r and s stones in them, respectively, you compute rs . Show that no matter how you split the piles, the sum of all the products computed at each step equals $\frac{n(n+1)}{2}$.