

## In-class, Week 6, day 1

Section 5.3, Problem 26: Let  $S$  be the subset of all ordered pairs of integers defined recursively by

- Basis Step:  $(0, 0) \in S$
  - Recursive Step: If  $(a, b) \in S$ , then  $(a+2, b+3) \in S$  and  $(a+3, b+2) \in S$ .
- a. List the elements of  $S$  produced by the first five applications of the recursive definition.
  - b. Use strong induction on the number of applications of the recursive step of the definition to show that  $5|a+b$  when  $(a, b) \in S$ .
  - c. Use structural induction to show that  $5|a+b$  when  $(a, b) \in S$ .

Section 5.3, Problem 32:

- a. Give a recursive definition of the function  $ones(s)$  which counts the number of ones in a bit string  $s$ .
- b. Use structural induction to prove that  $ones(st) = ones(s) + ones(t)$ .