COMP 2300 Midterm

May 2, 2012

This exam is closed-book. The work must be entirely entirely your own. A partial list of facts and definitions has been provided. Additional information may be provided at the discretion of the instructor. If in doubt, ask.

Name:_____

4-character code, if not already provided:_____

1. (10 points) Construct a truth table for the proposition $((p \vee q) \rightarrow \neg(p \wedge q)$. (10 points)

2. (5 points each)

   (a) Show that $\neg(p \rightarrow q) \rightarrow \neg q$ is a tautology without using a truth table.

   (b) Show that $(q \wedge \neg p) \vee p$ is equivalent to $q \vee p$ without using a truth table.

3. Determine the truth value of each of the statements below if the domain for all variables is the integers. (5 points each)

   (a) $\exists n(2n < n)$

   (b) $\forall n(n^2 > n)$

4. Determine the truth value of each of the statements below if the domain for all variables is the integers. (5 points each)

   (a) $\exists n \forall m(nm = m)$

   (b) $\exists n \exists m(n^2 + m^2 = 6)$

5. (5 points each)

   (a) Prove or disprove: If x is irrational then $\frac{1}{x}$ is irrational.

   (b) Prove that if n is an even integer, there exists an integer m such that n=(m+2)+(m+4).

6. (5 points each)

   (a) Prove that $\mathscr{P}(A) \subseteq \mathscr{P}(B) \leftrightarrow A \subseteq B$.

   (b) List the elements of $\{n \in \mathbb{Z}^+ | \exists k \in \mathbb{Z}(n = 3k)\} \cap \{n \in \mathbb{Z} | n < 20\}$.

7. (5 points each)

   (a) Represent the set $(A \cup B) - \left(A \cap C \cap \overline{B}\right)$ using a Venn Diagram.

   (b) Show $(A \cap B) \cup \left(A \cap \bar{B}\right) = A$ without using a Venn Diagram.

8. Give the power set of $\{\{2\}, \{2, \{2\}\}\}$. (10 points)

9. Determine whether each of the following functions from $\mathbb{Z}$ to $\mathbb{Z}$ is one-to-one. (5 points each)

   (a) $f(n) = n^3$

   (b) $f(n) = n^2 + 1$

10. In each of the following problems, specify two uncountable sets, A and B with the given property.

    (a) $A - B$ is finite. (3 points)

    (b) $A - B$ is countably infinite. (3 points)

    (c) $A - B$ is uncountable. (4 points)