COMP 2300: Name: ______________________

Discrete Structures Finals Points 30%

This exam is closed-book. All questions have equal weight. If a question has multiple parts, the weight assigned to that question is divided equally among all parts unless specified otherwise. The work must be entirely entirely your own. A partial list of facts and definitions has been provided. Additional information may be provided at the discretion of the instructor. If in doubt, ask.

1. Prove that $\sum_{i=1}^{n}(-1)^{i-1}i^2 = (-1)^{n-1}n(n+1)/2$ whenever n is a positive integer.

2. Prove that every amount of postage of 12 cents or more can be formed using just 4-cent and 5-cent stamps.

3. Let S be the set of strings defined recursively by

   - *basis step*: $1 \in S$
   - *Recursive step*: If $s \in S$ then $01s \in S$, $10s \in S$, $0s1 \in S$, $1s0 \in S$, $s10 \in S$, $s01 \in S$, $s1 \in S$, and $1s \in S$.

   (a) Prove that if $s \in S$ then there are more 1's than 0's in s.

   (b) Is the string 10011 in S? If so, show how to construct 10011 from the definition. If not, why not?

4. How many 6-digit numbers, not having 0 as the left-most digit

   (a) begin or end with the digit 9?

   (b) alternate between odd and even digits, for example 123638 but not 124638?

5. How many strings of length 10 contain at least three 1s and three 0s? Giving an answer without reasoning will only get you 50% of the grade.

6. If a camp has 12 cabins, what is the smallest number of campers that will guarantee that at least one cabin has more than 6 people?

7. What is the coefficient of x^3y^7 in $(3x - y)^{10}$?

8. What is the probability that a fair die never comes up an even number when it is rolled six times?

9. A standard die with six equally likely faces labeled 1,2,3,4,5, and 6 is rolled twice. The random variable X equals 1 if the two values rolled are equal, and 0 otherwise.

   (a) What is the expected value of X?

   (b) The random variable X is as defined above. The random variable Y equals 1 if the sum of the two values is 6, and 0 otherwise. What is the expected value of $X + Y$?

10. Suppose that 8% of all bicycle racers use steroids, that a bicyclist who uses steroids tests positive for steroids 96% of the time, and that a bicyclist who does not use steroids tests positive for steroids 9% of the time. What is the probability that randomly selected bicyclist who tests positive for steroids actually uses steroids?

11. *Extra Credit* Let S be the set of all ordered pairs of positive integers. For (a, b) and (c, d) in S, define $((a, b), (c, d)) \in \mathcal{R}$ if and only if $ad = bc$. Is $\mathcal{R}$ an equivalence relation? (*Hint* Think about fractions.)