## Normalization Highlights

Need to know from book:

- defintion of trivial FD
- defintion of partial FD
- defintion of transitive FD
- defintions of $1 \mathrm{NF}, 2 \mathrm{NF}, 3 \mathrm{NF}$, and BCNF and be able to understand when schemas are in which normal form.

Goal: BCNF, lossless, DP
If can not achieve goal, settle for 3NF, lossless, DP
Assume R is decomoposed into $R_{1}, R_{2}$. The decomposition is lossless if either:

- $R_{1} \cap R_{2} \rightarrow R_{1}$ or
- $R_{1} \cap R_{2} \rightarrow R_{2}$

Def: $F^{+}=$closure of $\mathrm{F}=$ set of all logically implied FDs

## Algorithm for determining closure of attr set $\alpha$ :

```
resultAttr = \alpha
resultFD = {}
while (changes to either result set)
{
\forallFD \beta}->\gamma\in
if \beta}\subset\mathrm{ resultAttr
then {
resultAttr = resultAttr }\cup
resultFd = resultFD ( }\beta->\gamma
} }
```

Example1:
Assume $R=\{A, B, C, D, E, F, G\}$ and $\mathrm{F}=\{B \rightarrow D, A \rightarrow B, C E \rightarrow F, C E \rightarrow G, A \rightarrow C\}$
Calculate $A^{+}$:

| iteration | resultAttr | resultFD |
| :--- | :--- | :--- |
| initial | A |  |
| 1 | $\mathrm{~A}, \mathrm{~B}$ | $A \rightarrow B$ |
| 1 | $\mathrm{~A}, \mathrm{~B}, \mathrm{C}$ | $A \rightarrow B, A \rightarrow C$ |
| 2 | A,B,C,D | $A \rightarrow B, A \rightarrow C, B \rightarrow D$ |
| 3 | no change | no change |

thus, $A^{+}=\{A, B, C, D\}\{A \rightarrow B, A \rightarrow C, B \rightarrow D\}$.

Example2:
$F=\{B \rightarrow D, A \rightarrow B, C E \rightarrow F, C E \rightarrow G, A \rightarrow C\}$
Calculate $A E^{+}$:

| iteration | resultAttr | resultFD |
| :--- | :--- | :--- |
| initial | A,E |  |
| 1 | A,B,E | $A \rightarrow B$ |
| 1 | A,B,C,E | $A \rightarrow B, A \rightarrow C$ |
| 2 | A,B,C,D,E | $A \rightarrow B, A \rightarrow C, B \rightarrow D$ |
| 2 | A,B,C,D,E,F | $A \rightarrow B, A \rightarrow C, B \rightarrow D, C E \rightarrow F$ |
| 2 | A,B,C,D,E,F,G | $A \rightarrow B, A \rightarrow C, B \rightarrow D, C E \rightarrow F, C E \rightarrow G$ |
| 3 | no change | no change |

thus, AE is a candidate key

## Minimal Cover

Before defining minimal cover, we need to define extraneous attribute.
Def: Attr A is extraneous in $\alpha, \alpha \rightarrow \beta$, if $A \in \alpha$ and $[(F-(\alpha \rightarrow \beta)) \cup((\alpha-A) \rightarrow \beta)]^{+} \equiv F^{+}$
Def: Attr A is extraneous in $\beta, \alpha \rightarrow \beta$, if $A \in \beta$ and $[(F-(\alpha \rightarrow \beta)) \cup(\alpha \rightarrow(\beta-A))]^{+} \equiv F^{+}$
Def: $F_{c}=$ minimal cover $=$ set of FD such that:

- $F_{c}^{+} \equiv F^{+}$
- no $\mathrm{FD} \in F_{c}$ contains extraneous attributes
- every $\mathrm{FD} \in F_{c}$ has a single value on the right hand side
- can not remove a FD from $F_{c}$ and stil have equivalance to $F^{+}$

Minimal Cover Construction Algorithm:

1. Set $G=F$
2. replace each FD, $X \rightarrow A_{1}, A_{2}, \ldots A_{n} \in G$ by FDs $X \rightarrow A_{1}, X \rightarrow A_{2}, \ldots X \rightarrow A_{n}$
3. $\forall \mathrm{FD} X \rightarrow A \in G$ : $\forall$ attribute B that is an element of $X$ : if $((G-\{X \rightarrow A\}) \cup\{(X-B) \rightarrow A\})$ is equivalent to $G$, then replace $X \rightarrow A$ with $(X-B) \rightarrow A$ in G .
4. $\forall \mathrm{FD} X \rightarrow A \in G$, compute $X^{+}$with respect to $\{G-(X \rightarrow A)\}$. If $A \in X^{+}$, remove $X \rightarrow A$ from G .

Note, you must do the steps in order. As your book shows, reversal of 3 and 4 may result in something that is NOT a minimal cover.

Example:
Let $F=\{A \rightarrow B C, B \rightarrow C, A \rightarrow B, A B \rightarrow C\}$. In step 2 we rewrite G as $\{A \rightarrow B, A \rightarrow$ $C, B \rightarrow C, A \rightarrow B, A B \rightarrow C\}$. There are no extraneous attributes to remove in step 3. In step 4 we remove the first $A \rightarrow B$, then remove $A \rightarrow C$, then finally remove $A B \rightarrow C$. Thus, $F_{c}=\{A \rightarrow B, B \rightarrow C\}$.

## Dependency Preservation

Consider the decomposition $R \Rightarrow R_{1}, R_{2}, \ldots R_{N}$.
Def: $F_{i}=$ restriction of F to $F_{i}=$ set of FDS that include only attributes of $R_{i}$.
Let $F^{\prime}=F_{1} \cup F_{2} \cup \ldots F_{N}$.
If $F^{\prime+} \equiv F^{+}$, then the decomposition is dependency preserving.
BCNF and Lossless Deomposition of $R$ Algorithm:

1. Set $D=\{R\}$
2. while $\exists$ schema $Q \in D$ that is not BCNF \{
(a) choose $Q \in D$ that is not BCNF
(b) find FD $X \rightarrow Y$ that violates BCNF
(c) replace $Q$ by the two schemas $(Q-Y)$ and $(X \cup Y)$
\}

Note, this decomposition is NOT guaranteed to be dependency preserving.
3NF, Lossless, and Dependency Preserving Deomposition of $R$ Algorithm:

1. Find a minimal cover $F_{c}$ for $F$
2. For each left hand side $X \in F_{c}$, create a relation scheme $\left(X \cup A_{1} \cup A_{2} \cup \ldots A_{N}\right)$ where $X \rightarrow A_{1}, X \rightarrow A_{2}, \ldots X \rightarrow A_{N}$ are all the dependencies in $F_{c}$ with $X$ in the left hand side.
