This homework is worth 10% of your grade. The points are equally distributed among the following problems. The questions are not ordered according to their difficulty. Undergraduates need not solve questions marked with asterisks (*).

1. Let a k-PDA be a pushdown automaton that has k stacks. Thus a 0-PDA is an NFA and a 1-PDA is a conventional PDA. You already know that 1-PDAs are more powerful (recognize a larger class of languages) than 0-PDAs.
   (a) Show that 2-PDAs are more powerful than 1-PDAs.
   (b) Show that 3-PDAs are not more powerful than 2-PDAs. (Hint: Simulate a Turing machine tape with two stacks.)
   (c) Describe the slowdown experienced by simulating a Turing machine by two stacks.
2. A *queue automaton* is like a push-down automaton except that the stack is replaced by a queue. A *queue* is a tape allowing symbols to be written only on the left-hand end and read only at the right-hand end. Each write operation (we'll call it a *push*) adds a symbol to the left-hand end of the queue and each read operation (we'll call it a *pull*) reads and removes a symbol at the right-hand end. As with a PDA, the input is placed on a separate read-only input tape, and the head on the input tape can move only from left to right. The input tape contains a cell with a blank symbol following the input, so that the end of the input can be detected. A queue automaton accepts its input by entering a special accept state at any time. Show that a language can be recognized by a deterministic queue automaton iff the language is Turing-recognizable.
3. Let $L = \{\langle A \rangle : A \text{ is a DFA and } L(A) = \Sigma^*\}$. Show that L is decidable.
4. Show that there exists a language over a unary alphabet that is Turing-recognizable but not Turing-decidable.
5. Let A and B be Turing-recognizable languages such that $A \bigcup B = \Sigma^*$. Show that there exists a decidable language C such that $\overline{B} \subseteq C \subseteq A$.
6. Let C be a language. Prove that C is Turing-recognizable iff a decidable language D exists such that $C = \{x\colon \exists y,\ \langle x, y \rangle \in D\}$.
7. Show that for any alphabet Σ there is a language L over Σ such that neither L nor $\Sigma^* - L$ is Turing-acceptable.

* 8. Show that if a Turing machine M accepts each string w in $L(M)$ after executing no more than $|w| + 1$ steps, then $L(M)$ must be regular.

* 9. Prove that the following decision problem is decidable.

   Given an arbitrary CFG G and an arbitrary regular expression r as input, decide whether or not $L(G) \subseteq L(r)$.

* 10. Show that for CFL L, it is undecidable whether $L = L^R$.