Calculus II Section 1: Dr. Judd
Lab 2: Riemann Sums
25 points, due Wednesday 13 February in class.

The assignment is due at the start of class Wednesday 13 February. It is your responsibility to download the lab and work on it in a timely manner.

The only way to submit the lab is by printing it out and bringing it to class. I will not accept the electronic file in an email, on a floppy disk, or any other way. You may work with your classmates on this assignment, but you should produce your own document on your own computer. Taking someone’s document and changing the name and a word or two violates our integrity policy.

Part I

The goal of this lab is to use Scientific Notebook to calculate Riemann sums. Recall that to find a Riemann sum of a function \( f \) over an interval \([a, b]\) one must first choose the number of rectangles or sub-intervals, say \( n \), and next partition the interval \([a, b]\) into \( n \) sub-intervals. For this project we shall use sub-intervals of equal length.

Suppose we want to calculate a Riemann sum of \( y = \ln(x) \) over the interval \([1, 10]\).

1.1 We decide to have 9 sub-intervals. If each sub-interval has the same length, what will be that length?

1.2 In general if we want to partition the interval \([a, b]\) into \( n \) sub-intervals of equal length, what will be the length of each sub-interval?

Now we must find the sub-intervals themselves. We must choose \( n - 1 \) points \( a < x_1 < x_2 < \cdots < x_{n-1} < b \) and set \( x_0 = a, x_n = b \).

1.3 If we partition \([1, 10]\) into 9 sub-intervals of equal length, then \( x_0 = 1 \) and \( x_9 = 10 \). What are \( x_1, x_2, \ldots, x_8 \)?

1.4 In general if we partition \([a, b]\) into \( n \) sub-intervals of equal length, then \( x_0 = a \) and \( x_n = b \). What are \( x_1, x_2, \ldots, x_{n-1} \)?

The next step is to choose a point \( c_k \) in \([x_{k-1}, x_k]\) for each \( k = 1, 2, \ldots, n \). A general Riemann sum allows us to pick any \( c_k \) in the sub-interval, but to make calculations easy we shall be more specific.

A **Left Riemann sum** uses the **left endpoint** of each interval for each \( c_k \), so \( c_1 = x_0, c_2 = x_1, \ldots, c_n = x_{n-1} \).

A **Right Riemann sum** uses the **right endpoint** of each interval for each \( c_k \), so \( c_1 = x_1, c_2 = x_2, \ldots, c_n = x_n \).

A **Midpoint Riemann sum** uses the **midpoint** of each interval for each \( c_k \).

1.5 What is the midpoint of \([4, 5]\)?

1.6 What is the midpoint of \([x_{k-1}, x_k]\)?

1.7 Write down values \( c_k \) \( (k = 1, 2, \ldots, 9) \) for Left, Right and Midpoint Riemann sums over \([1, 10]\) using the sub-intervals from (1.3).
Recall that if \( P = \{x_0, x_1, \ldots, x_n\} \) is a partition of \([a, b]\), \( \Delta_k = x_k - x_{k-1} \) is the length of the \( k^{th} \) sub-interval and \( c_k \) is in \([x_{k-1}, x_k]\) for \( k = 1, 2, \ldots, n \), then the corresponding Riemann sum is

\[
\sum_{k=1}^{n} f(c_k) \Delta_k .
\]

In our case each sub-interval has the same length \( \Delta \) and so the sum becomes

\[
\sum_{k=1}^{n} f(c_k) \Delta \quad \text{or, more simply,} \quad \Delta \sum_{k=1}^{n} f(c_k) .
\]

1.8 Show why \( \sum_{k=1}^{n} a_k \Delta = \Delta \sum_{k=1}^{n} a_k \).

1.9 Calculate Left, Right and Midpoint Riemann sums for \( f(x) = \ln x \) on the interval \([1, 10]\) using 9 sub-intervals of equal length.

1.10 Use Scientific Notebook to calculate the definite integral

\[
\int_{1}^{10} \ln x \, dx .
\]

How do your answers to (1.9) compare with this integral?

Part II

Construct and calculate midpoint Riemann sums for the following functions over the given intervals. Determine how many equal length intervals are required to find the answer correct to 3 decimal places (and show why).

2.1 \( f(x) = e^x/\sqrt{x^4-1} \) over the interval \([-2, 2]\).

2.2 \( f(x) = \sin x \) over the interval \([0, 2\pi]\).

2.3 \( f(x) = \sqrt{9-x^2} \) over the interval \([-3, 3]\).

We know that if \( f(x) \geq 0 \) on \([a, b]\), then \( \int_{a}^{b} f(x) \, dx \) is the area between the curve \( y = f(x) \) and the \( x \)-axis from \( a \) to \( b \).

2.4 What curve is \( y = \sqrt{9-x^2} \)?

2.5 What should be the area \( \int_{-3}^{3} \sqrt{9-x^2} \, dx \) and why?

2.6 Is this the same answer you found in (2.3)?

Robert Judd, 1 February 2002.