5.4 First Order Separable Differential Equations

Goals.
• Know what “First Order Separable Differential Equation” means.
• Check when a function is a solution of a given differential equation.
• Solve first order separable differential equations.

Problems.
1. Show that \( y = \frac{(\cos x)}{x} \) is a solution of the differential equation \( x \frac{dy}{dx} + y = -\sin x, \ x > 0, \) with initial condition \( y(\pi/2) = 0. \)
2. Solve \( \frac{dy}{dx} = \sqrt{y} \cos^2 \sqrt{y}. \)
3. Solve \( x \frac{dy}{dx} = y \ln x. \)
4. Solve \( \sin x - (y \cos^2 x) \frac{dy}{dx} = 0. \)

Chapter 6 Exponentials and Logarithms

Goals.
• Know the integral definition of the natural logarithm.
• Know the definition of \( e^x \) in terms of \( \ln x. \)
• Know the definition of the hyperbolic trigonometric functions.
• Be able to differentiate, integrate and manipulate the hyperbolic trig. functions.

Problems.
1. State the integral definition of the natural logarithm.
2. Define \( \cosh x, \sinh x \) and \( \tanh x. \)
3. What is the equivalent of \( \cos^2 x + \sin^2 x = 1 \) for \( \cosh x \) and \( \sinh x? \)
4. Simplify \( \ln(\cosh x + \sinh x) + \ln(\cosh x - \sinh x). \)
5. Find the derivative of \( y = \ln(\cosh x) - 2 \tanh^2 x \) with respect to \( x. \)
6. Find the following integrals:
\[
\begin{align*}
\int_0^{\ln 2} 4 \cosh(3x - \ln 2) \, dx & \quad \int_{\ln 2}^{\tan h(x/7)} \, dx & \quad \int_0^{\coth(x/\sqrt{3})} \, dx \\
\int_0^{\ln 2} \tanh 2x \, dx & \quad \int_1^2 \frac{\cosh(\ln x)}{x} \, dx & \quad \int_0^0 \cosh^2(x/2) \, dx
\end{align*}
\]