7.2 Integration by Parts

Goals.
- Be able to integrate by parts
- Apply integration by parts to definite and indefinite integrals
- Use substitution with integration by parts
- Remember that “1” is a perfectly good function to use as “u”.

Problems.
1. Write down the formula for integration by parts
2. Find the following integrals:
   a. \( \int x^3 \ln x \, dx \)
   b. \( \int \sin^{-1}x \, dx \)
   c. \( \int x^2 e^{-x} \, dx \)
   d. \( \int_{\pi/2}^{0} x^2 \cos 2x \, dx \)
   e. \( \int_{0}^{1} x\sqrt{1-x} \, dx \)
   f. \( \int x(\ln x)^2 \, dx \)
   g. \( \int \cos xe^{-x} \, dx \)
   h. \( \int \ln 2x \, dx \)

7.3 Partial Fractions

Goals.
- Be able to decompose a rational function into partial fractions
- Integrate rational functions using partial fractions
- Apply partial fractions when the numerator has greater degree than the denominator
- Be able to handle the cases where the denominator has repeated linear terms, or quadratic terms, when it has been completely factored

Problems.
1. Expand the following using partial fractions:
   a. \( \frac{5x - 7}{x^2 - 3x + 2} \)
   b. \( \frac{x}{x^3 - x^2 - 6x} \)
   c. \( \frac{x^4 + 9}{x^4 + 9x^2} \)
   d. \( \frac{x^3}{x^2 - 4x + 4} \)
   e. \( \frac{x^2}{(x - 1)(x^2 + 2x + 1)} \)
   f. \( \frac{3x^2 + x + 4}{x^3 + x} \)
   g. \( \frac{x^2 + 2x + 1}{(x^2 + 1)^2} \)
   h. \( \frac{1}{x^4 - 2x^2 + 1} \)
2. Find the following integrals:
   a. \( \int \frac{5x - 7}{x^2 - 3x + 2} \, dx \)
   b. \( \int_{0}^{2} \frac{x}{x^3 - x^2 - 6x} \, dx \)
   c. \( \int \frac{x^4 + 9}{x^4 + 9x^2} \, dx \)
   d. \( \int_{-1}^{1} \frac{x^3}{x^2 - 4x + 4} \, dx \)
   e. \( \int \frac{x^2}{(x - 1)(x^2 + 2x + 1)} \, dx \)
   f. \( \int_{-1}^{2} \frac{3x^2 + x + 4}{x^3 + x} \, dx \)
   g. \( \int \frac{x^2 + 2x + 1}{(x^2 + 1)^2} \, dx \)
   h. \( \int \frac{1}{x^4 - 2x^2 + 1} \, dx \)