7.1 A Nightmare of Integrals

The general idea of this section is to be able to integrate almost anything. Here are some techniques you should know:

Goals.
- Substitution
- Completing the square
- Trigonometric identities
- Eliminating square roots
- Reducing an improper fraction (polynomial division)
- Separating a fraction
- Multiplying by “1” (typically this mean multiplying top and bottom of a fraction by the same thing, or by something like $\cos^2 x + \sin^2 x = 1$).

There are examples of all of these in the text, section 7.1.

Problems. Calculate the following integrals:

1. $\int \frac{1}{x^2 - 4x + 8} \, dx$
2. $\int \cos^2 3x \, dx$
3. $\int \frac{2x - 1}{x^2 + 4} \, dx$
4. $\int \frac{x + 2}{\sqrt{4 - x^2}} \, dx$
5. $\int \sin^3 x \, dx$
6. $\int \frac{1}{2 \sin x \cos x} \, dx$
7. $\int e^x \cos 2x \, dx$
8. $\int x^2 \sin(1 - x) \, dx$
9. $\int \frac{1}{e^x - 1} \, dx$

7.4 Trigonometric Substitution

Goals.
- Know the basic trigonometric substitutions and how to use them.
- Know the valid domains/ranges for $x$ and $u$ in each substitution.

Problems. Find the following integrals:

1. $\int \frac{x}{\sqrt{16 - x^2}} \, dx$
2. $\int \frac{x}{\sqrt{4 + x^2}} \, dx$
3. $\int \frac{x}{4 - x^2} \, dx$
4. $\int \frac{x}{\sqrt{4x^2 - 1}} \, dx$
5. $\int \frac{1}{\sqrt{e^{2x} - 1}} \, dx$
6. $\int \frac{8}{x\sqrt{49x^2 - 4}} \, dx$