Calculus III: Dr. Judd
Review for Test 1.

The test will cover Chapter 7, sections 4, 6 and 7, and Chapter 8, section 1. This review sheet is not meant to be an exhaustive list of problems that will be on the test, but rather a sample of things to expect. I strongly recommend that you look up my tests from Calculus I and II (on the web site) so that you can see what to expect from the test.

7.4 Trigonometric Substitution

Know the basic trigonometric substitutions and how to use them. Know the valid domains/ranges for $x$ and $u$ in each substitution.

Find the following integrals:

1. $\int \frac{x}{\sqrt{16 - x^2}} \, dx$
2. $\int \frac{x}{\sqrt{4 + x^2}} \, dx$
3. $\int \frac{x}{4 - x^2} \, dx$
4. $\int \frac{x}{\sqrt{4x^2 - 1}} \, dx$
5. $\int \frac{1}{\sqrt{ex^2 - 1}} \, dx$
6. $\int \frac{8}{x\sqrt{49x^2 - 4}} \, dx$

7.6 L’Hôpital’s Rule

Know the indeterminate forms to which l’Hôpital’s rule may be applied. Know how to apply l’Hôpital’s rule. Be able to re-arrange the forms "$\infty - \infty$" and "$\infty \cdot 0$" in order to apply l’Hôpital’s rule. Understand how to take the logarithm through the limit. Understand that $\infty$ is not a real number and hence trying to assign a value such as 0 or 1 to expressions like "$\infty \cdot 0$" or "$1^\infty$" is meaningless.

Evaluate the following limits:

1. $\lim_{x \to 0} \frac{\sin 7x}{\tan 11x}$
2. $\lim_{x \to \infty} \frac{e^x + x^2}{e^x - x}$
3. $\lim_{x \to 1^+} x^{1/(1-x)}$
4. $\lim_{x \to \infty} x \tan(1/x)$
5. $\lim_{x \to 0^+} \frac{1}{x} - \frac{1}{\sqrt{x}}$
6. $\lim_{x \to 0} \left( \frac{1}{x^2} \right)^x$
7. $\lim_{x \to \infty} \frac{\sqrt{9x + 1}}{\sqrt{x + 1}}$
8. $\lim_{x \to 0} \frac{1 - \cos x}{x^2}$
9. $\lim_{x \to 0} \frac{\sin x}{\cos x}$
10. $\lim_{x \to \infty} \left( 1 + \frac{3}{x} \right)^x$

Find functions $f$ and $g$ such that $\lim_{x \to 0} f(x) = 0$, $\lim_{x \to 0} g(x)$ diverges to infinity, and

0.11 $\lim_{x \to 0} f(x)/g(x) = 3$
0.12 $\lim_{x \to 0} f(x)/g(x) = -17$
0.13 $\lim_{x \to 0} f(x)/g(x) = 0$
0.14 $\lim_{x \to 0} f(x)/g(x)$ diverges to infinity
0.15 $\lim_{x \to 0} f(x)/g(x)$ diverges to negative infinity

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7.7 Improper Integrals

Know what an improper integral is. Be able to evaluate improper integrals. Know what it means for an integral to converge or diverge. Understand why it is incorrect (and will receive no points on the test) to attempt to evaluate an improper integral without using a limit.

1. \[ \int_{2}^{\infty} \frac{3}{x^2 - x} \, dx \]
2. \[ \int_{1}^{\infty} \frac{1}{x\sqrt{x^2 - 1}} \, dx \]
3. \[ \int_{0}^{1} \frac{1}{\sqrt{1 - x^2}} \, dx \]
4. \[ \int_{0}^{2} \frac{1}{1 - x^2} \, dx \]
5. \[ \int_{-\infty}^{0} xe^x \, dx \]
6. \[ \int_{1}^{0} \ln x \, dx \]
7. \[ \int_{1}^{1} \frac{1}{x^{2/3}} \, dx \]

8.1 Sequences

Understand the idea of sequences. Know and be able to use the definition of convergence for a sequence. Know the properties of limits of convergent sequences. Be able to find the limit of a convergent sequence using the definition; the Sandwich Theorem; continuous functions; and l’Hôpital’s rule.

Determine whether the following sequences are convergent or divergent. If the sequence converges find its limit, otherwise state the manner in which it diverges.

1. \( a_n = \ln n - \ln(n + 1) \)
2. \( a_n = \left(\frac{1}{3}\right)^n + \frac{1}{\sqrt{2^n}} \)
3. \( a_n = \tan^{-1} n \)
4. \( a_n = \left(1 - \frac{1}{n}\right)^n \)
5. \( a_n = \ln \left[\left(1 - \frac{1}{n}\right)^n\right] \)
6. \( a_n = \frac{n!}{10^n} \)
7. \( a_n = \sqrt[4]{4^n} \)
8. \( a_n = \frac{\ln n}{n^{1/n}} \)
9. \( a_n = e^{-n} \cos n \)

Do the same for these sequences:

1. \( a_n = 1 + (-1)^n \)
2. \( a_n = \left(\frac{n + 1}{2n}\right) \left(1 - \frac{1}{n}\right) \)
3. \( a_n = \frac{\sin n}{n} \)
4. \( a_n = \frac{n}{2^n} \)
5. \( a_n = \frac{\sin^2 n}{2^n} \)
6. \( a_n = \frac{\ln(n + 1)}{\sqrt{n}} \)
7. \( a_n = \frac{n!}{n^n} \)
8. \( a_n = \left(\frac{n}{n + 1}\right)^n \)
9. \( a_n = \sqrt[n^2 + n]}{\sqrt[n^2 + n]} \)