MATH 3451 Homework Assignment 3

Instructions: Solve and turn in all of the assigned problems, taken from our textbook. Problems marked with a * must be done by graduate students, and may be attempted by undergraduates for extra credit.

Due on Thursday, October 10th at the beginning of class.

Section 1.5 (page 38): 9

• If $\mu > 2 + \sqrt{5}$, prove that every periodic point for $f_\mu = \mu x(1-x)$ is repelling. (You can use facts from class!)

For the remaining problems, you can/should use symbolic codings. As a reminder, I_0 and I_1 are the two closed intervals making up $[0,1] \setminus A_0$. If $x \in \Lambda$, then the symbolic coding of x is a 0-1 sequence $.x_0x_1x_2\ldots$, where $x_n = 0$ if $f^n x \in I_0$ and $x_n = 1$ if $f^n x \in I_1$. For example, the symbolic coding of 1 is $.100000\ldots$, since $1 \in I_1$, but $f^n(1) = 0 \in I_0$ for all $n > 0$.

• If $\mu > 5$, prove that for every n, if $x, y \in \Lambda$ and the symbolic codings of x and y agree on the first n bits, then $|x - y| < 2^{-n}$. (Hint: Mean Value Theorem, used similarly to how we showed that Λ contains no intervals!)

For the following two problems, use the previous problem, and the fact we 'proved' in class: for every 0-1 sequence, there exists $x \in \Lambda$ with that symbolic coding.

• If $\mu > 5$, use the previous problem to show that for any 0-1 sequence, there exists only ONE point $x \in \Lambda$ with that symbolic coding.

• If $\mu > 5$, use the problem before the previous problem to show that there exists a point $x \in \Lambda$ so that 0 is not in the orbit of x, but 0 is a limit of some subsequence of the orbit of x, i.e. there exists an increasing sequence (n_k) so that $f^{n_k}x \to 0$.