MATH 3451 Homework Assignment 5

Instructions: Solve and turn in all of the assigned problems.

Due on Thursday, October 31st at the beginning of class.

1. Show that if (X, f^2) is topologically transitive, then (X, f) is as well.

2. If (X, f) is topologically transitive and $X \subseteq \mathbb{R}$, show that for any nonempty open U, V, and W, there exist $n > m > 0$ and $x \in U$ where $f^m x \in V$ and $f^n x \in W$. (HINT: Consider sets of the form $V \cap f^{-k} W$!)

3. Show that if f is strictly increasing on $\mathbb{R}$, then $(\mathbb{R}, f)$ is not topologically transitive.

4. Define Σ'' to be the set of all sequences of 1, 2, and 3, where no two consecutive digits are equal. For instance, $123123123 \ldots \in \Sigma''$, but $1232112321 \ldots \notin \Sigma''$. Define Σ''' to be the set of all sequences of 1, 2, and 3 where each digit is at least as large as the previous one. For instance, $111223333333 \ldots \in \Sigma'''$, but $1112232 \ldots \notin \Sigma'''$.
   (a) Show that the periodic points of Σ'' are dense.
   (b) Show that the periodic points of Σ''' are not dense.

5. Prove that if f is C^1 on $\mathbb{R}$ and has a fixed point x with $|f'(x)| < 1$, then $(\mathbb{R}, f)$ is NOT sensitive to initial conditions.