MATH 3451 Homework Assignment 6

Instructions: Solve and turn in all of the assigned problems.

Due on Thursday, November 14th at the beginning of class.

1. We proved the following fact in class and called it Lemma 1: if $I \to J$, then there exists $I' \subseteq I$ with $f(I') = J$. Use that fact to prove the following, which we called Lemma 2: if $I_0 \to I_1 \to I_2 \cdots \to I_n$, then there exists $I_0' \subseteq I_0$ for which $f^i(I_0) \subseteq I_i$ for $0 \leq i \leq n$ and $f^n(I_0) = I_n$.

2. Give an example of a function f with points of every least period.

3. Prove the following facts about the relationship between least periods of f^2 and least periods of f: (these will be very useful for problems 3 and 4!)

• If a point x has least period $2n$ for f, then x has least period n for f^2.
• If a point x has least period n for f^2 and n is EVEN, then x has least period $2n$ for f.
• If a point x has least period n for f^2 and n is ODD, then x has least period either $2n$ for f or least period n for f.

4. Use induction on n to prove Sharkovsky's Theorem for all k of the form $2^n j$, j odd and greater than 1, from the proof of Sharkovsky's Theorem for k odd and greater than 1 given in class.

5. Use induction on n to prove Sharkovsky's Theorem for all k of the form 2^n by using the fact, proved in class, that any f with a non-fixed periodic point contains a point of least period 2.