MATH 3451 Final Exam

1. (a) State the three properties that a dynamical system must have in order to satisfy Devaney's definition of chaos.

(b) Explain why $f(x) = \frac{x}{2}$ on $\mathbb{R}$ has NONE of these properties.

2. For the function $f(x) = x(1-x)$, explain, with proof, the behavior of the orbit $f^n(x_0)$ as $n \to \infty$ for every possible x_0. (You may use, without proof, properties given by the graph of $f(x)$ on the board.)

3. (a) For the function $f(x) = 5x(1-x)$, define the symbolic coding $s : \Lambda \to \Sigma_2$. You do not have to prove any of its properties, but explain how $s(x)$ is defined for any $x \in \Lambda$.

(b) Use part (a) to explain why there exists a point whose orbit is dense in Λ.

4. Prove that if a continuous function f on $\mathbb{R}$ has a periodic point, then it has a fixed point. Do NOT use Sharkovsky's theorem to prove this.

5. For each of the following, give an example of a continuous function f on $\mathbb{R}$ with the desired properties or explain why such an example cannot exist. You DO NOT have to prove that your example has the desired properties.

(a) f has points of least period 1 and 2, but no others

(b) f has points of least period 1, 2, and 3, but no others

(c) f has points of all least periods EXCEPT 3.

6. (a) In your own words, describe the two types of bifurcations which we discussed in class for a family $f_\mu(x)$. You do not have to state the hypotheses under which the bifurcations occur, but try to clearly state what happens as μ passes the value where the bifurcation occurs.

(b) For the family $f(x) = \mu \ln x$, both types of bifurcations occur exactly once. Find the two values of μ where these occur, via any technique that you wish.