MATH 3451 Review Sheet for Final Exam

The final exam will be comprehensive (i.e. anything from the course could be asked about), and it will probably be about half questions from pre-midterm material and half questions from post-midterm material. The sections of Devaney we've covered are 1.1-1.8, 1.10, and 1.12 (though we only very briefly treated SDIC in 1.8 and bifurcations in 1.12). The test will consist of some shorter problems which relate to definitions/computations and will not require you to show proof (one-third to one-half of the total points on the test), and some proof-based problems.

Here's a list of definitions and types of problems from our course. I've attempted to list as many as possible, but this list is not meant to be completely comprehensive.

Terms with which you should be familiar: open, closed, dense, continuous, homeomorphism, orbit, fixed, periodic, eventually fixed, eventually periodic, least period, stable set, hyperbolic/attracting/repelling periodic point, topologically conjugate, Cantor set, quadratic map (i.e. $f(x) = \mu x(1-x)$), symbolic coding (for the quadratic map, when $\mu > 4$), shift map (i.e. σ), topologically transitive, Sharkovsky's Theorem, tangent bifurcation, period-doubling bifurcation. (SDIC is not here because it will not be represented on the exam.)

Each of the following bullet points WILL be represented on the test somewhere:

- Using our standard arguments to describe limiting behavior of orbits for a function $f : \mathbb{R} \to \mathbb{R}$. Our usual technique is to use fixed points to split into intervals where $f(x) > x$ or $f(x) < x$, then for each interval to either do an induction argument to show that the orbit is increasing/decreasing and find its limit or to show that the interval is mapped to a different interval where such an induction argument can be applied. (Though it's too long for the exam, look at the solution to Written Problem 2 on HW2 to see an example of this type of argument.)

- Be familiar with the topological conjugacy $S : \Lambda \to \Sigma_2$ given by the symbolic coding for the quadratic map when $\mu > 4$, and know how to use it to prove that points exist in Λ with certain behaviors. (See the solution to Written Problem 2 from HW4.)

- Be prepared to do a problem related to the least periods that points may have for a continuous function $f : \mathbb{R} \to \mathbb{R}$. This could, for instance, involve using Sharkovsky's theorem to give statements about which least periods f does or does not have, or it could involve using our various lemmas to prove that points with various least periods exist for a specific f WITHOUT applying Sharkovsky's theorem. (See problems 4 and 5 from the 2015 final exam.)