MATC 1101  
Analytical Inquiry I  
Project 4  
Due: Tuesday, February 24, 2004.

In the following problems, show enough detail to indicate how you obtained your answer.

1. Convert the base 2 number 101011 to base 10.

2. Convert 51 to base 2 using the “highest power of 2” algorithm described on page 35 of the course notes.

3. Convert 51 to base 2 using the “division/remainder” algorithm described on page 36 of the course notes.

4. How is 51 stored in memory when considered as the two characters 5 and 1? (See the ASCII Character Table under links on our web page.)

5. (a) Convert the following base 8 number to base 10:

   435.

(b) Convert the following base 10 number to base 8:

   222.

   Hint: try an algorithm similar to the ones for going from base 10 to base 2.

6. Find the truth tables (using 0s and 1s) for each of the following logical statements.

   (a) $p \& \&(\neg p)$.
   (b) $\neg(p \| q)$.
   (c) $\neg(p \& q)$.
   (d) $p \& \&(q \| r)$. (Do this one for the eight ways of assigning 0 or 1 to $p$, $q$, and $r$.

7. Show that the full adder in Figure 2 of the supplement obtains $s$ and $c$ for all combinations of $x$, $y$, and $z$ in the table on page 4 of the supplement.

8. Give an electrical circuit (in terms of half and full adders) for a three-bit adder that performs the addition:

   $a_3a_2a_1$
   $b_3b_2b_1$
   $s_4s_3s_2s_1$

1
9. (a) Subtract the following base two numbers:

\[ 1101 - 111. \]

(b) The half subtractor operation is defined by the following table:

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
<th>d</th>
<th>b</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

where \( d \) is the difference digit and \( b \) is the borrow bit. For example

\[
\begin{align*}
1 - 1 & = 0 = d, \text{ with } b = 0, \\
1 - 0 & = 1 = d, \text{ with } b = 0, \\
0 - 1 & = 1 = d, \text{ with } b = 1, \\
0 - 0 & = 0 = d, \text{ with } b = 0.
\end{align*}
\]

Notice the table assumes that \( 0 - 1 \) requires a borrow bit.

i. Give logical formulas (involving the logical operators &&, ||, and !) for \( d \) and \( b \).

ii. Give an electrical circuit for the half-subtractor.