This homework is worth 10% of your grade. The points are equally distributed among the following problems. The questions are not ordered according to their difficulty. Undergraduates need not solve questions marked with asterisks (*).

1. (Question 7 from the previous homework) Give a regular expression for

\[ L = \{ w : w \text{ has an even } \# \text{ of } 0\text{s AND an odd } \# \text{ of } 1\text{s} \} \]

(Hint: Build a DFA and convert it to a regular expression)

2. Use the construction given in Theorem 1.39 to convert the following nondeterministic finite automaton to an equivalent deterministic finite automaton.

3. Use the procedure described in Lemma 1.55 to convert the following regular expressions to nondeterministic finite automata.

(a) \((0 \cup 1)^*000(0 \cup 1)^*\)
(b) \(((00)^*(11) \cup 01)^*\)
(c) \(\emptyset^*\)

4. Use the procedure described in Lemma 1.60 to convert the following finite automaton to a regular expression.

5. Use the pumping lemma to show that the following language is not regular.

\[ L = \{ w w w : w \in \{ a, b \}^* \} \]

6. For any string \( w = w_1 w_2 \cdots w_n \), the reverse of \( w \), written \( w^R \), is the string \( w \) in reverse order, \( w_n \cdots w_2 w_1 \). For any language \( A \), let \( A^R = \{ w^R : w \in A \} \). Show that if \( A \) is regular, so is \( A^R \).
7. Is $L = \{ww^R \mid w, u \in (a+b)^*\}$ regular? Justify your answer, i.e. if you answer yes, give an automaton; otherwise, prove that $L$ is not regular.

* 8. For languages $L_1$ and $L_2$, define

$$L_1 \ominus L_2 = \{w \mid w \in L_1 \text{ and } w \text{ does not contain any string from } L_2 \text{ as a substring}\}.$$ 

Prove that if $L_1$ and $L_2$ are regular, then so is $L_1 \ominus L_2$.

* 9. One can construct a regular expression from a finite automaton by solving a set of linear equations of the form

$$
\begin{pmatrix}
    x_1 \\
    x_2 \\
    \vdots \\
    x_n
\end{pmatrix} =
\begin{pmatrix}
    a_{11} & a_{12} & \cdots & a_{1n} \\
    a_{21} & a_{22} & \cdots & a_{2n} \\
    \vdots & \vdots & \ddots & \vdots \\
    a_{n1} & a_{n2} & \cdots & a_{nm}
\end{pmatrix}
\begin{pmatrix}
    x_1 \\
    x_2 \\
    \vdots \\
    x_n
\end{pmatrix} +
\begin{pmatrix}
    c_1 \\
    c_2 \\
    \vdots \\
    c_n
\end{pmatrix},
$$

where $a_{ij}$ and $c_i$ are sets of string denoted by regular expressions, $+$ denotes set union, and multiplication denotes concatenation. Give an algorithm for solving such equations.

* 10. Use the ideas that show how to convert an NFA to regular expression to construct algorithms for the following problems.

(a) Find the lowest-cost path between two vertices in a directed graph where each edge is labeled with a nonnegative cost.

(b) Determine the number of strings of length $n$ accepted by a finite automaton.