**Discontinuities**

We take a look at the various types of discontinuity and how they occur.

**Removable Discontinuity**

A removable discontinuity is a point \( x = c \) where the function has a discontinuity, but may be redefined at that point to make it continuous.

\[
f(x) = \frac{x^2 - 1}{x - 1}
\]

![Graph of f(x) = \(\frac{x^2 - 1}{x - 1}\)]

You can see that this function has a discontinuity at \( x = 1 \) (in fact this point is not in the domain of \( f \)) however if we define \( f(1) = 2 \), then this becomes a continuous function.

**Jump Discontinuity**

This is where the function has a jump either side of the point \( x = c \).
The function $f(x)$ has the given graph. It has a jump discontinuity at $x = 0$. Notice that there is no way to redefine $f$ at $x = 0$ to make it continuous as in the previous example.

**Infinite Discontinuity**

A function has an infinite discontinuity if the limit at $x = c$ is plus or minus infinity.

$f(x) = \frac{1}{x^2}$

$f(x) = \frac{1}{x}$
Oscillating Discontinuity

An oscillating discontinuity occurs when the value of the function is changing so rapidly that a limit is not possible. The classic example is

\[ f(x) = \sin(1/x) \]
Exercises

On what intervals are the following functions continuous? Beware of removable discontinuities which this program will ignore!

1. 
   \[ f(x) = \frac{x + 1}{x^2 - 4x + 3} \]

2. 
   \[ f(x) = \frac{1}{|x| + 1} - \frac{x^2}{2} \]

3. 
   \[ f(x) = \frac{\cos x}{x} \]

Are the following functions continuous at the given point?

4. \( \sin(x = \sin x) \) at \( x = \pi \)
5. \( \sin(\frac{\pi}{2} \cos(\tan x)) \) at \( x = 0 \)
6. \( \sec(x \sec^2 x - \tan^2 x - 1) \) at \( x = 1 \)