Inverse Functions and Logarithms

Graphs of functions and their inverses

In each of the following pairs of graphs the original function is red, its inverse is blue and the line \( y = x \) is black. See how the graph of the inverse is the reflection of the graph of the function in the line \( y = x \).

\[
f(x) = x^2, \text{ with domain } [0, \infty) \text{ and range } [0, \infty) \\
f^{-1}(x) = \sqrt{x}, \text{ with domain } [0, \infty) \text{ and range } [0, \infty) \\
\]

\[
g(x) = \frac{2x - 1}{x + 1}, \text{ domain } (x \neq -1), \text{ range } (y \neq 2) \\
g^{-1}(x) = -\frac{x + 1}{x - 2}, \text{ domain } (x \neq 2), \text{ range } (y \neq -1) \\
\]
\( h(x) = 2^{x/4}, \text{ domain } (-\infty, \infty), \text{ range } (0, \infty) \)

\( h^{-1}(x) = 4 \ln x / \ln 2 \)

\( x = 2^{y/4}, \text{ Solution is: } \{ y = 4 \frac{\ln x}{\ln 2} \} \)

In order to use Scientific Notebook to find the inverse of a function, say \( y = (2x - 1)/(x + 1) \), you can simply swap \( x \) and \( y \) as usual, and then use Compute/Solve/Exact (or the solve button).

- Enter \( x = (2y - 1)/(y + 1) \)
- Click Compute/Solve/Exact next to the equation and when prompted ask to solve for \( y \) to obtain: \( \{ y = \frac{x+1}{x-2} \} \)