Sample Midterm Exam

Name: _____

Read all of the following information before starting the exam:

- READ EACH OF THE PROBLEMS OF THE EXAM CAREFULLY!

- Show all work, clearly and in order, if you want to get full credit. I reserve the right to take off points if I cannot see how you arrived at your answer (even if your final answer is correct).

- A single 8 1/2 × 11 sheet of notes (double sided) is allowed. No calculators are permitted.

- Circle or otherwise indicate your final answers.

- Please keep your written answers clear, concise and to the poin.

- This test has xxx problems and is worth xxx points. It is your responsibility to make sure that you have all of the pages!

- Turn off cellphones, etc.

- Good luck!

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1. (*20 points*) The Fibonacci sequence is $\{f_n\} = 1, 1, 2, 3, 5, 8, 13, 21, ...$ is defined so that $f_n = f_{n-1} + f_{n-2}$; that is each term of the sequence is the sum of the two previous terms.

a. (*5 pts*)   Does $\{f_n\}$ converge or diverge?

b. (*15 pts*)   Mathematician Abraham de Moivre found an exact formula for the nth term of the Fibonacci sequence, which is

$$f_n = \frac{1}{\sqrt{5}} \left(\frac{1 + \sqrt{5}}{2} \right)^n - \frac{1}{\sqrt{5}} \left(\frac{1 - \sqrt{5}}{2} \right)^n.$$

Use this to show that the sequence

$$a_n = \frac{f_n}{\left(\frac{1+\sqrt{5}}{2} \right)^n}$$

converges, and find the limit.

2. (*20 points*) Determine whether the following series converge or diverge. Indicate how you know.

  a. (*10 pts*)

$$\sum_{n=1}^{\infty} 3^{-\ln(n)}$$

  b. (*10 pts*)

$$\sum_{k=1}^{\infty} \frac{x^2 + 3}{x^2 - 1}$$

3. (*20 points*) Use the integral test to determine whether or not

$$\sum_{i=10}^{\infty} \frac{1}{i \ln(i) \ln \ln(i) \ln \ln \ln(i)}.$$

is convergent or divergent. *HINT: Try letting $u = \ln \ln \ln(x)$.*

4. (*20 points*) For each statement, mark it true of false. If it is false give a (counter)example. If it is true give a reason - if the reason is a theorem, state the theorem, otherwise give a brief proof. No credit for answers without a correct reason or example.

a. (*3 pts*)   If $\lim_{n\to\infty} a_i = 0$ then $\sum_{i=1}^{\infty} a_i$ converges.

b. (*3 pts*)

$$\int_1^{x^y} \frac{1}{t} dt = y \int_1^x \frac{1}{t} dt$$

c. (*3 pts*)   If $\sum_{i=1}^{\infty} a_i$ converges, then all $a_i > 0$.

d. (*4 pts*)   If $\{a_i\}$ is a monotone sequence then it is increasing.

e. (*4 pts*)   If $\{a_i\}$ is convergent, then it is bounded and monotone.

f. (*3 pts*)   The integral $\int_0^\infty x^p dx$ diverges for all p.

5. (*20 points*)

a. (*10 pts*) Find an explicit formula for the following sequence $\{a_n\} = 1, -\frac{1}{2}, \frac{1}{4}, -\frac{1}{8}, \ldots$. What type of sequence is this?

b. (*10 pts*) Find the value of the series $\sum_{i=1}^{\infty} a_i$ if it converges, where $\{a_i\}$ from part (*a*). If it diverges, state how you know.

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