Sample Midterm Exam

Math 112Z **Name:** ________________________________
9/28/08

Read all of the following information before starting the exam:

- READ EACH OF THE PROBLEMS OF THE EXAM CAREFULLY!
- Show all work, clearly and in order, if you want to get full credit. I reserve the right to take off points if I cannot see how you arrived at your answer (even if your final answer is correct).
- A single 8 1/2 $\times$ 11 sheet of notes (double sided) is allowed. No calculators are permitted.
- Circle or otherwise indicate your final answers.
- Please keep your written answers clear, concise and to the point.
- This test has xxx problems and is worth xxx points. It is your responsibility to make sure that you have all of the pages!
- Turn off cellphones, etc.
- Good luck!

1	
2	
3	
4	
5	
$\sum$	

1. (*20 points*) Determine whether the following series converge absolutely, converge conditionally or diverge. **a.** (*10 pts*)

$$\sum_{n=1}^{\infty}(-1)^n \frac{3n+2}{n^2}.$$

b. (*10 pts*)

$$\sum_{n=1}^{\infty}\left(\frac{3n+2}{2n+1}\right)^n.$$

2. (*20 points*) Determine the radius and interval of convergence for the following power series.

a. (*10 pts*)

$$\sum_{n=0}^{\infty} \frac{x^{2n}}{3^n}$$

b. (*10 pts*)

$$\sum_{n=1}^{\infty} \frac{e^n x^n}{n^n}.$$

3. (*20 points*)

a. (*10 pts*) Give an example of a series that has both positive and negative terms, but is not alternating.

b. (*10 pts*) Consider the alternating series

$$\sum_{n=1}^{\infty}(-1)^n \frac{1}{(n+1)^2}.$$

How many terms must be used to estimate the sum accurately within $\frac{1}{10000}$?

4. (*20 points*) For each statement, mark it true of false. If it is false give a (counter)example. If it is true give a reason - if the reason is a theorem, state the theorem, otherwise give a brief proof. No credit for answers without a correct reason or example.

a. (*3 pts*) A power series $\sum_{n=0}^{\infty} c_n x^n$ can converge on the interval $(0, \infty)$.

b. (*3 pts*) If an alternating series has decreasing terms, then it converges.

c. (*3 pts*) If $\sum_{n=0}^{\infty} c_n x^n$ converges for $x = 4$, then it converges for $x = -2$.

d. (*4 pts*) If $f(x) = \sum_{n=0}^{\infty} c_n(x-a)^n$ is differentiable on its interval of convergence, then a power series for $f'(x)$ is

$$f'(x) = \sum_{n=}^{\infty} nc_n(x-a)^n.$$

e. (*4 pts*)

$$\sum_{n=0}^{\infty} x^n = \frac{1}{1-x}$$

for all x.

f. (*3 pts*)

$$\sum_{n=0}^{\infty} \frac{x^n}{n!} = e^x$$

for all x.

5. (*20 points*)

a. (*10 pts*) Starting with the geometric series: $\sum_{i=0}^{\infty} x^n$, find the sum of the series $\sum_{i=0}^{\infty} nx^{n-1}$ for $|x| < 1$.

b. (*10 pts*) Starting with the power series representation for $\frac{1}{1-x}$ find the power series representation for $\frac{1}{2-x}$. What is the interval of convergence of the series you find?