Sample Midterm Exam

Math 112Z
9/28/08

Name: ______________________________

Read all of the following information before starting the exam:

- READ EACH OF THE PROBLEMS OF THE EXAM CAREFULLY!
- Show all work, clearly and in order, if you want to get full credit. I reserve the right to take off points if I cannot see how you arrived at your answer (even if your final answer is correct).
- A single 8 1/2 $\times$ 11 sheet of notes (double sided) is allowed. No calculators are permitted.
- Circle or otherwise indicate your final answers.
- Please keep your written answers clear, concise and to the point.
- This test has xxx problems and is worth xxx points. It is your responsibility to make sure that you have all of the pages!
- Turn off cellphones, etc.
- Good luck!

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$\sum$	

1. (*20 points*) Determine whether the following series converge absolutely, converge conditionally or diverge.

a. (*10 pts*)

$$\sum_{n=1}^{\infty}(-1)^n\frac{n}{n^{3/2}+1}.$$

b. (*10 pts*)

$$\sum_{n=1}^{\infty}\frac{(n!)^2}{((2n)!)}$$

2. (*20 points*) Determine the radius and interval of convergence for the following power series.

a. (*10 pts*)

$$\sum_{n=1}^{\infty} (\ln n)^n x^n$$

b. (*10 pts*)

$$\sum_{n=0}^{\infty} \left(\frac{2n+3}{n+2} \right)^{n^2} x^{n^2}$$

3. (*20 points*) Consider the power series:

$$f(x) = \sum_{n=1}^{\infty} (-1)^n \frac{x^n}{ne^n}$$

a. (*10 pts*) Find the radius and interval of convergence for the power series.

b. (*5 pts*) Find a power series representation for $f'(x)$. What is the interval and radius of convergence for this new power series?

c. (*5 pts*) Find a power series representation for $f(x^2)$. What is the interval and radius of convergence for this new power series?

4. (*20 points*) Give an example of each of the following: **a.** (*5 pts*) A power series with interval of convergence $(0, 2]$.

b. (*5 pts*) A power series with radius of convergence $R = \infty$.

c. (*5 pts*) A series which is absolutely convergence, but is not alternating or strictly positive.

d. (*5 pts*) Two series such that $f_n < g_n$ for all n, $\sum_{n=0}^{\infty} f_n$ diverges and $\sum_{n=0}^{\infty} g_n$ converges. (Hint: what hypothesis of the comparison test is missing?

5. (*20 points*)
a. (*10 pts*) Use the power series for $\ln(1-x)$ to find a power series for $\ln(x)$. What is the radius and interval of convergence for this power series?

b. (*10 pts*) Note that $e^{-1} = \sum_{n=0}^{\infty} \frac{(-1)^n}{n!}$. How many terms must be used to estimate e^{-1} with an error of at most $\frac{1}{120}$. (If you can not solve explicitly for n, just leave an expression, but it is set up to have a nice answer.)