Sample Midterm Exam

Math 112Z
9/28/08

Name: _____

Read all of the following information before starting the exam:

- READ EACH OF THE PROBLEMS OF THE EXAM CAREFULLY!
- Show all work, clearly and in order, if you want to get full credit. I reserve the right to take off points if I cannot see how you arrived at your answer (even if your final answer is correct).
- A single 8 1/2 × 11 sheet of notes (double sided) is allowed. No calculators are permitted.
- Circle or otherwise indicate your final answers.
- Please keep your written answers clear, concise and to the point.
- This test has xxx problems and is worth xxx points. It is your responsibility to make sure that you have all of the pages!
- Turn off cellphones, etc.
- Good luck!

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$\sum$	

1. (*20 points*) **a.** (*10 pts*) One way of expressing the function $\sinh(x)$ is

$$\sinh(x) = \frac{e^x - e^{-x}}{2}.$$

Use this and the Maclaurin series for e^x to find a Maclaurin series for $\sinh(x)$.

b. (*10 pts*) If $f(x) = \sinh(x)$ find a series representation for $f'(x^2)$.

2. (*20 points*)

a. (*10 pts*) Suppose $f(x)$ is an increasing concave up function at $x = 0$. Is it possible that

$$f(x) = 3 + 2x - x^2 + .3x^4 + 0.02x^5 + \dots.$$

Why or why not?

b. (*10 pts*) Suppose $f(x) = \sum_{n=1}^{\infty} \frac{1}{n} x^n$. What is $f^{10}(0)$?

3. (*20 points*) Using the Maclaurin series for $\cos(x)$, and e^x find the first three non-zero terms of the Maclaurin series for $\cos(x)e^x$.

4. (*20 points*) **a.** (*10 pts*) Suppose the Maclaurin series for $f(x)$ starts:

$$f(x) = 1 - \frac{1}{2}x - \frac{1}{10}x^2 + \dots$$

and we know that $f^{(3)}(x) < \frac{1}{100}$ for $|x| < 1$. Use Taylor's inequality to bound the error in the statement

$$f(0.1) \approx 1 - \frac{1}{2}(0.1) + \frac{1}{10}(0.1)^2.$$

b. (*10 pts*) Use series representations for cos(x), sin(x) to find

$$\lim_{x \to 0} \frac{\cos(x) - 1}{x \sin(x)}$$

5. (*20 points*)
a. (*10 pts*) Find

$$\int \arcsin(x)dx$$

Hint: Try integration by parts.

b. (*10 pts*) Find

$$\int \frac{4}{(x-1)(x^2+1)}.$$