Sample Final Exam

Math 112Z
9/28/08

Name: _____

Read all of the following information before starting the exam:

- READ EACH OF THE PROBLEMS OF THE EXAM CAREFULLY!
- Show all work, clearly and in order, if you want to get full credit. I reserve the right to take off points if I cannot see how you arrived at your answer (even if your final answer is correct).
- A single 8 1/2 $\times$ 11 sheet of notes (double sided) is allowed. No calculators are permitted.
- Circle or otherwise indicate your final answers.
- Please keep your written answers clear, concise and to the point.
- This test has xxx problems and is worth xxx points. It is your responsibility to make sure that you have all of the pages!
- Turn off cellphones, etc.
- Good luck!

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$\sum$	

1. (*20 points*)

a. (*10 pts*) Give an example of a *sequence* which is convergent, and one that is divergent. Clearly label which is which.

b. (*10 pts*) Determine the limit of the following sequence, if it exists:

$$a_n = \frac{\sqrt{1+n}-1}{n}.$$

2. (*20 points*) Determine whether the following series converge or diverge. If they converge find the sum:

a. (*5 pts*)

$$\sum_{n=0}^{\infty} \frac{e^n}{2^{2n}}.$$

b. (*5 pts*)

$$\sum_{k=1}^{\infty} \frac{\sqrt{k}+1}{k^{3/2}}.$$

c. (*10 pts*)

$$\sum_{n=0}^{\infty} \frac{(-1)^n \pi^{2n}}{4^{2n}(2n)!}.$$

3. (*20 points*)

a. (*10 pts*) Sketch the polar curve

$$r = \cos(3\theta).$$

b. (*10 pts*) Find the area contained within one petal of the curve:

$$r = \cos(3\theta).$$

4. (*20 points*)

a. (*10 pts*) Simplify the following expression by combining all the terms:

$$\frac{1}{2}\ln(a) + 2\ln(b) - 3\ln(c^2).$$

b. (*10 pts*) State the definition of $\ln(x)$ as an integral.

5. (*20 points*)

a. (*10 pts*) Suppose

$$f(x) = \sum_{n=0}^{\infty} \frac{1}{2n^2} x^n.$$

Find the radius and interval of convergence of this series. *Be sure to check the endpoints!*

b. (*10 pts*) Suppose

$$g(x) = \sum_{n=0}^{\infty} n(x-2)^n.$$

What is $f^{(1000)}(2)$?

6. (*20 points*)

a. (*10 pts*) Find the area contained within the circle defined by the polar curve

$$r = 2\cos(\theta).$$

Hint: If you don't get π^2 you are doing something wrong. See part b. Think: what is wrong with my limits. You may even want to sketch the curve.

b. (*10 pts*) The polar curve defined in part (a) is a circle of radius one. Explain (in words) why

$$\int_0^{2\pi} \frac{1}{2} r^2 d\theta = 2\pi^2,$$

as opposed to π^2 (which is the area of the circle.)

7. (*10 points*)

a. (*10 pts*) Find an equation for the following polar curve in standard coordinates (that is, describe the curve in terms of x, y):

$$r = 4\sin(\theta).$$

Hint: Start by multiplying both sides by r.

b. (*10 pts*) What is the polar equation describing a circle of radius π centered at the origin?

8. (*20 points*)

a. (*10 pts*) Does $\sum_{n=1}^{\infty}(-1)^n \frac{1}{n^2}$ converge absolutely, conditionally or neither?

b. (*10 pts*) Use the comparison or limit comparison test to determine the convergence of

$$\sum_{n=1}^{\infty} \frac{n^3+2n-1}{n^5-n+10}.$$

9. (*20 points*) **a.** (*10 pts*) Estimate the error incurred by using the first 10 terms of

$$\sum_{i=1}^{\infty}(-1)^n\frac{1}{n^3}$$

as an estimate for the infinite series.

b. (*10 pts*) Estimate the error incurred by using the first 10 terms of

$$\sum_{i=1}^{\infty}\frac{1}{n^3}$$

as an estimate for the infinite series. *Hint: Recall that this error is bounded by an integral!*

10. (*20 points*) **a.** (*10 pts*) Find a Taylor series centered at 2 for

$$f(x) = x^3 - 3x^2 + 1.$$

Note: The Taylor series you find will be a finite polynomial.

b. (*10 pts*) Find

$$\int_0^{infty} xe^{-x}dx.$$

Note: be careful with the limits when computing this (improper) integral!

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