

Math 361, Problem set 9

Due 11/8/10

1. (2.5.3) Let $p(x_1, x_2) = \frac{1}{16}$, $x_1 = 1, 2, 3, 4$ and $x_2 = 1, 2, 3, 4$, zero elsewhere, be the joint pmf of X_1 and X_2 . Show that X_1, X_2 are independent.
2. (2.5.8) Let X and Y have the joint pdf $f(x, y) = 3x$, $0 < y < x < 1$, zero elsewhere. Are X and Y independent? If not, find $\mathbb{E}[X|y]$.
3. (2.5.9) Suppose that a man leaves for work between 8 : 00 and 8 : 30 AM, and takes between 40 and 50 minutes to get to the office. Let X denote the time of departure and Y denote the time of travel. If we assume that these random variables are independent and uniformly distributed, find the probability that he arrives at the office before 9:00 AM.
4. (2.6.1) Let X, Y, Z have the joint pdf $f(x, y, z) = \frac{2(x+y+z)}{3}$, $0 < x < 1$, $0 < y < 1$, $0 < z < 1$, zero elsewhere.
 - Find the marginal probability density functions of X , Y , and Z .
 - Compute $\mathbb{P}(0 < X < 1/2, 0 < Y < 1/2, 0 < Z < 1/2)$ and $\mathbb{P}(0 < X < 1/2)$.
 - Are X , Y and Z independent.
 - Calculate $\mathbb{E}[X^2YZ + 3XY^4Z^2]$
 - Determine the cdf of X , Y , and Z .
 - Find the conditional distribution of X and Y , given $Z = z$ and evaluate $\mathbb{E}[X + Y|z]$.
 - Determine the conditional distribution of X given $Y = y$ and $Z = z$, and compute $\mathbb{E}[X|y, z]$.
5. (2.7.3) Let X_1, X_2, X_3 be iid with common pdf $f(x) = e^{-x}$, $x \geq 0$ 0 elsewhere. Find the joint pdf of $Y_1 = X_1, Y_2 = X_1 + X_2$ and $Y_3 = X_1 + X_2 + X_3$.
6. (3.1.25) Let X_1 and X_2 have binomial distribution with parameters $n_1, p_1 = \frac{1}{2}$ and $n_2, p_2 = \frac{1}{2}$ respectively. Show that $Y = X_1 - X_2 + n_2$ has a binomial distribution with parameters $n = n_1 + n_2, p = \frac{1}{2}$.