Math 361, Problem set 5

Due 10/04/10

1. (1.6.8) Let X have the pmf $p(x) = (\frac{1}{2})^x$, $x = 1, 2, 3, \dots$, and zero elsewhere. Find the pmf of $Y = X^3$.

2. (a) Pick a card from a standard deck. Let X denote the rank of the card(counting ace as one, J=11, Q=12, K=13.) Find the pmf of X.

   (b) Pick two cards from a deck, with replacement. Let Y denote the highest rank picked. Find the pmf of Y.

3. (1.7.8) A mode of a distribution of one random variable X is a value of x that maximizes the pdf or pmf. For X of the continuous type, $f(x)$ must be continuous. If there is only one such x, it is called the mode of the distribution. Find the mode of each of the following distributions:

   (a) $p(x) = (\frac{1}{2})^x$, $x, 1, 2, 3, \dots$, zero elsewhere.

   (b) $f(x) = 12x^2(1 - x), 0 < x < 1$, zero elsewhere.

   (c) $f(x) = \frac{1}{2}x^2 e^{-x}$, $0 < x < \infty$, zero elsewhere.

4. (1.7.14) Let X have the pdf $f(x) = 2x$, $0 < x < 1$, zero elsewhere. Compute the probability that X is at least $\frac{3}{4}$ given that X is at least $\frac{1}{2}$.

5. (1.7.17) Divide a line segment into two parts by selecting a point at random. Find teh probability that the larger segment is at least 3 times the shorter. Assume the point is chosen uniformly.

6. (1.7.22) Let X have the uniform pdf $f_X(x) = \frac{1}{\pi}$ for $-\frac{\pi}{2} < x < \frac{\pi}{2}$. Find the pdf of $Y = \tan(X)$. This is the pdf of a **Cauchy distribution**.