Math 361, Problem set 6

Due 10/18/10

1. (1.8.3) Let X have pdf $f(x+2)/18$ for $-2 < x < 4$, zero elsewhere. Find $\mathbb{E}[X], \mathbb{E}[(X+2)^3]$ and $\mathbb{E}[6X - 2(X+2)^3]$.

2. (1.8.5) Let X be a number selected uniformly random from a set of numbers $\{51, \ldots, 100\}$. Approximate $\mathbb{E}[1/X]$. *Hint: Find reasonable upper and loewr bounds by finding integrals bounding* $\mathbb{E}[1/X]$.

3. Let X have the pdf $f(x) = 1/x^3$. Find $\mathbb{E}[X]$, but show that $e[X^2]$ does not exist.

4. (1.8.14) Let Xhave the pdf $f(x) = 3x^2$, $0 < x < 1$, zero elsewhere.

   (a) Compute $\mathbb{E}[X^3]$

   (b) Show that $Y = X^3$ has a uniform(0,1) distribution.

   (c) Compute $\mathbb{E}[Y]$ and compate this result with the answer obtained in Part (a).

5. (1.9.4) If the $\mathbb{E}[X^2]$ exists, show that

$$\mathbb{E}[X^2] \geq (\mathbb{E}[X])^2$$

6. (1.9.8) Let X be a random variable such that $\mathbb{E}[(X-b)]$ exists for all real b. Show that $\mathbb{E}[(X-b)^2]$ is minimized when $b = \mathbb{E}[X]$.