

Math 361, Problem set 9

Due 11/1/10

1. (2.2.3) Let X_1 and X_2 have the joint pdf $h(x_1, x_2) = 2e^{-x_1-x_2}$, $0 < x_1 < x_2 < \infty$, zero elsewhere. Find the joint pdf of $Y_1 = 2X_1$ and $Y_2 = X_2 - X_1$.
2. (2.3.2) Let $f_{1|2}(x_1|x_2) = c_1x_1/x_2^2$, $0 < x_1 < x_2$, $0 < x_2 < 1$ zero elsewhere, and $f_2(x_2) = c_2x_2^4$, $0 < x_2 < 1$, zero elsewhere, denote, respectively, the conditional pdf of X_1 given $X_2 = x_2$ and the marginal pdf of X_2 . Determine
 - (a) The constants c_1 and c_2 .
 - (b) The joint pdf of X_1 and X_2 .
 - (c) $\mathbb{P}(\frac{1}{4} < X_1 < 1/2 | X_2 = \frac{5}{8})$
 - (d) $\mathbb{P}(1/4 < X_1 < 1/2)$
3. (2.3.5) Let X_1 and X_2 be two random variables such that the conditional distributions and means exist. Show that
 - (a) $\mathbb{E}[X_1 + X_2 | X_2] = \mathbb{E}[X_1 | X_2] + X_2$
 - (b) $\mathbb{E}[u(X_2) | X_2] = u(X_2)$.
4. (2.3.9) Five cards are drawn at random and without replacement from an ordinary deck of cards. Let X_1 and X_2 denote, respectively, the number of spades and the number of hearts that appear in the five cards.
 - (a) Determine the joint pmf of X_1 and X_2
 - (b) Find the two marginal pmfs
 - (c) What is the conditional pmf of X_2 given $X_1 = x_1$.

Note: First two parts are similar to what was on your last homework!

5. (2.3.11) Let us choose at random a point from the interval $(0, 1)$ and let the random variable X_1 be equal to the number which corresponds to that point. Then choose a point at random from the interval $(0, x_1)$, where x_1 is the experimental value of X_1 ; and let the random variable X_2 be equal to the number which corresponds to this point.

- (a) Make assumptions about the marginal pdf $f_1(x_1)$ and the conditional pdf $f_{2|1}(x_2|x_1)$.
- (b) Compute $\mathbb{P}(X_1 + X_2 \geq 1)$.
- (c) Find the conditional mean $\mathbb{E}[X_1|x_2]$.