1. (20 points) There are two urns. The first has 3 red balls and 5 blue balls, the second has 6 red balls and 100 blue balls. A coin is flipped. If heads comes up a uniform random ball is picked from the first urn, if tails comes up a uniform random ball is picked from the second. Given that the picked ball is blue, what is the conditional probability that the coin was a heads.

H=event win heads B=event ball blue

Want:
$$P(B|H)P(H)$$

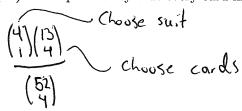
$$P(B|H)P(H) + P(B|T)P(T)$$

$$= \frac{5}{8} \cdot \frac{1}{2}$$

$$= \frac{53}{133}$$

$$\frac{5}{8} \cdot \frac{1}{2} + \frac{100}{106} \cdot \frac{1}{2}$$
Okay answer,
we need to simplify to

2. (20 points) A hand of 4 cards is picked from a standard deck. Compute a. (7 pts) the probability that every card in the hand is of the same suit.



- - Every card in the hand is a different suit and a different rank.

- **3.** (20 points) X is a continuous random variable with pdf $f(x) = 2e^{-2x}$ if $x \ge 0$, and f(x) = 0 otherwise. Compute
 - **a.** (7 pts) $\mathbb{P}(X \in C_1)$ where $C_1 = \mathbb{N} = \{1, 2, 3, 4, ...\}.$

$$P(X=1)=P(X=2)=...0$$
 as $\int_{0}^{q} 2e^{-2x} dx=0$.
 $P(X\in C_{i})=\sum_{k=1}^{q} P(X=k)=0$.

b.
$$(7 pts)$$
 $\mathbb{P}(X \ge 3)$

$$\int_{3}^{\infty} 2e^{-2x} dx = -e^{-2x} \Big|_{3}^{\infty} = e^{-6}$$

c.
$$(6 \text{ pts})$$
 $\mathbb{P}(X \ge 3 | X \ge 2)$.

 $P(X \ge 7) - \int_{2}^{\infty} e^{-7x} dx = -e^{-7x} \int_{2}^{\infty$

- 4. (20 points) Two fair dice are rolled.
- a. (10 pts) Let X denote the absolute value of the difference between the rolls. Compute the pmf p(x) of X.

×	$\rho(x)$
0	36
1	1936
2	8/36
3 4	6/36
7 5	2/36
els &	0

b. (10 pts) Determine the probability that the product of the two rolls is greater than the sum of the two rolls.

Easier: Product & Sum: Only happens if at least one of rolls is one, or both are two.

 $P(Product \leq Sum) = \frac{12}{36} = \frac{1}{3}$ $P(Product > Sum) = \frac{2}{3}$

(1,1) (2,1) (1,6) (6,1) 12 poss. U/ Product & Sum.

5. (20 points) a. (10 pts) Could $F(x) = \frac{1}{2}^x$ for x = 0, 1, 2, 3... denote the cdf of a random variable X. Why or why not?

F(x) is decreasing, could not be CDF.

(or, at least,
not increasing)

b. (10 pts) A continuous random variable X has pdf $f(x) = \frac{1}{x^2}$ for $x \ge 1$, 0 otherwise. Compute the cdf and pdf of $Y = X^2$.

Y = g(x) $g(x) = \sqrt{x}$ $(g^{-1})^{1}(x) = 2\sqrt{x}$ $f(g^{-1})^{1}(y) = (\frac{1}{2y \cdot \sqrt{y}})$ $(g^{-1})^{1}(y) = (\frac{1}{2y \cdot \sqrt{y}})$