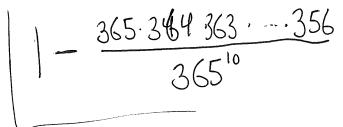
1. (20 points) a. (10 pts) 10 people are in a room. Assuming their birthdays are independent and uniformly distributed, what is the probability that at least two of them share a birthday.



b. (10 pts) How many people must be in a room so that the probability that at least two of them share a birthday is 0.5?

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- **2.** (20 points) Recall: The mode of a random variable X is the value of x where the pmf p(x) or pdf $f_X(x)$ is maximized (depending on whether the random variable is continuous.) Find the mode of the following random variables.
 - **a.** (8 pts) X is discrete with pmf $p(x) = e^{-10} \frac{10^x}{x!}$ for x = 0, 1, 2, 3, ...

$$\frac{\rho(x+1)}{\rho(x)} = \frac{e^{-10}10^{x+1}}{(x+1)!} = \frac{10}{x+1}.$$

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b. (7 pts) X is continuous with pdf $f_X(x) = \sin(x)$, $0 \le x \le \pi$ $f_X(x) = 0$ otherwise.

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$$2$$

a pole 2
 $dx f(x) = \frac{\cos x}{2}$; zero at $x = \frac{\pi}{2}$.
Mode is $x = \frac{\pi}{2}$

c. (7 pts) X is continuous with pdf $f_X(x) = \frac{1}{e-1}e^x$ for $0 \le x \le 1$, $f_X(x) = 0$ otherwise.

3. (20 points) a. (10 pts) Construct an example showing that 3 events can be pairwise independent, but not mutually independent.

b. (10 pts) Suppose A, B, and C are mutually independent with $\mathbb{P}(A) = 1/2$, $\mathbb{P}(B) = 1/4$, $\mathbb{P}(C) = 3/8$. Find

$$P((AUB^c) \cap C^c) = P(AUB^c) P(C^c)$$

= $(P(A) + P(B^c) - P(A) P(B^c)) P(C^c)$
= $(\frac{1}{2} + \frac{3}{4} - \frac{3}{8})(\frac{5}{8}) = \frac{35}{64}$

4. (20 points) A random word is chosen unformly from the sentence

"How much wood could a woodchuck chuck if a woodchuck could chuck wood."

Let X denote the length of the word.

- a. (10 pts) Find the pmf of X $\times \gamma \ell \lambda$ 1 | 2/3 2 | 1/3
- 3 1/3
- 5 4/3
- 60
- 80
- 9 2/13

b. (10 pts) Suppose two words are chosen uniformly from the sentence, with replacement. What is the probability that they have the same length?

$$P(Some | enoth) = (\frac{2}{13})^{2} + (\frac{1}{13})^{2} + (\frac{1}{13})^{2} + (\frac{3}{13})^{2} + (\frac{4}{13})^{2} + (\frac{2}{13})^{2}$$

$$\frac{1}{13}$$

$$\frac{1}{13}$$

$$\frac{1}{13}$$

$$\frac{1}{13}$$

$$\frac{1}{13}$$

$$\frac{1}{13}$$

$$\frac{1}{13}$$

$$\frac{1}{13}$$

$$\frac{1}{13}$$

Note: Without replacement is

$$\frac{(2)(1)}{(3)(1)} + \frac{(3)(2)}{(3)(1)} + \frac{(4)(3)}{(3)(1)} + \frac{(2)(1)}{(3)(1)}$$

both 1 both 9.

5. (20 points) a. (10 pts) Suppose X is discrete with pmf $p(x) = \frac{6}{\pi^2} \cdot \frac{1}{x^2}$, p(x) = 0 otherwise. Find the pmf of $Y = X^2$.

Y=
$$g(X)$$
 where $g'(X)=J_X$

$$P_{\chi}(y)=P_{\chi}(J_y)=\int_{X}^{G} \frac{dy}{dx} \cdot \frac{dy}$$

b. (10 pts) Suppose X if continuous with pdf $f(x) = \frac{1}{x^2}$ for $x \ge 1$, f(x) = 0 otherwise. Find the pdf of $Y = X^2$.