Midterm Exam I

Math 361 **Name:** _____
9/27/10

Read all of the following information before starting the exam:

- READ EACH OF THE PROBLEMS OF THE EXAM CAREFULLY!
- Show all work, clearly and in order, if you want to get full credit. I reserve the right to take off points if I cannot see how you arrived at your answer (even if your final answer is correct).
- A single 8 1/2 × 11 sheet of notes (double sided) is allowed. Calculators are permitted.
- Circle or otherwise indicate your final answers.
- Please keep your written answers clear, concise and to the point.
- This test has . problems and is worth 100 points. It is your responsibility to make sure that you have all of the pages!
- Turn off cellphones, etc.
- READ EACH OF THE PROBLEMS OF THE EXAM CAREFULLY!
- Good luck!

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1. (*20 points*) **a.** (*10 pts*) 10 people are in a room. Assuming their birthdays are independent and uniformly distributed, what is the probability that at least two of them share a birthday.

b. (*10 pts*) How many people must be in a room so that the probability that at least two of them share a birthday is 0.5?

2. (*20 points*) Recall: The mode of a random variable X is the value of x where the pmf $p(x)$ or pdf $f_X(x)$ is maximized (depending on whether the random variable is continuous.) Find the mode of the following random variables.

a. (*8 pts*) X is discrete with pmf $p(x) = e^{-10}\frac{10^x}{x!}$ for $x = 0, 1, 2, 3, \ldots$.

b. (*7 pts*) X is continuous with pdf $f_X(x) = \sin(x)$, $0 \leq x \leq \pi$ $f_X(x) = 0$ otherwise.

c. (*7 pts*) X is continuous with pdf $f_X(x) = \frac{1}{e-1}e^x$ for $0 \leq x \leq 1$, $f_X(x) = 0$ otherwise.

3. (*20 points*) **a.** (*10 pts*) Construct an example showing that 3 events can be pairwise independent, but not mutually independent.

b. (*10 pts*) Suppose A, B, and C are mutually independent with $\mathbb{P}(\mathrm{A}) = 1/2$, $\mathbb{P}(\mathrm{B}) = 1/4$, $\mathbb{P}(\mathrm{C}) = 3/8$. Find

$$\mathbb{P}((\mathrm{A} \cup \mathrm{B}^{\mathrm{c}}) \cap \mathrm{C}^{\mathrm{c}})$$

4. (*20 points*) A random word is chosen unformly from the sentence

"How much wood could a woodchuck chuck if a woodchuck could chuck wood."

Let X denote the length of the word.

a. (*10 pts*) Find the pmf of X

b. (*10 pts*) Suppose two words are chosen uniformly from the sentence, with replacement. What is the probability that they have the same length?

5. (*20 points*) **a.** (*10 pts*) Suppose X is discrete with pmf $p(x) = \frac{6}{\pi^2} \cdot \frac{1}{x^2}$, $p(x) = 0$ otherwise. Find the pmf of $Y = X^2$.

b. (*10 pts*) Suppose X if continuous with pdf $f(x) = \frac{1}{x^2}$ for $x \geq 1$, $f(x) = 0$ otherwise. Find the pdf of $Y = X^2$.

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