Practice Midterm Exam I

Math 362 2/25/10

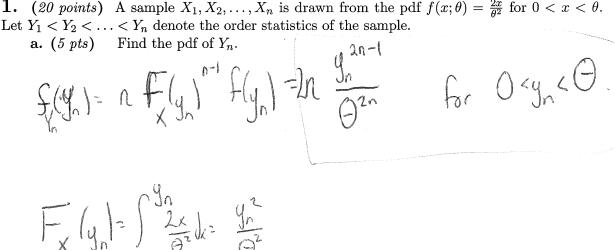
Name: _____

Read all of the following information before starting the exam:

- READ EACH OF THE PROBLEMS OF THE EXAM CAREFULLY!
- Show all work, clearly and in order, if you want to get full credit. I reserve the right to take off points if I cannot see how you arrived at your answer (even if your final answer is correct).
- \bullet A single 8 1/2 \times 11 sheet of notes (double sided) is allowed. Calculators are permitted.
- Copies of normal, t-distribution and χ^2 tables are at the back
- Circle or otherwise indicate your final answers.
- Please keep your written answers clear, concise and to the point.
- This test has . problems and is worth 100 points. It is your responsibility to make sure that you have all of the pages!
- Turn off cellphones, etc.
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- Good luck!

Practice 1 Solns

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5	
\sum	



b. (7 pts) Find c that makes
$$cY_n$$
 an unbiased estimator for θ .

$$\begin{aligned}
& \left[\left[\begin{array}{c} V \\ 1 \end{array} \right] = \int_{0}^{1} \frac{\partial}{\partial x^{2n}} = \frac{2n}{2^{n+1}} \frac{\partial}{\partial x^{2n}} = \frac{2n$$

Suppose n = 5, and $\theta = 1$. Find the probability that Y_n is a potential outlier **c.** (8 pts) for its distribution.

for its distribution.

$$f_{Y_5}(y) = 10$$
 Q_{9} $Q_{$

2. (20 points) X_1, \ldots, X_n are independent Poisson random variables with mean λ . We wish to test

$$H_0: \lambda = \frac{1}{10}$$
 vs $H_1: \lambda > \frac{1}{10}$.

a. (10 pts) Suppose n = 10. We accept H_1 if $Y = \sum X_i > C$. How large must C be in order for the size of the test to be less than .05?

b. (10 pts) Another possible test is the following: We accept H_1 if more than 1/10 of the observed values are non-zero. Find the approximate size of this test. You may leave your answer in terms of Φ , the cdf of the standard normal distribution and n. Do not assume n = 10. Hint: The number of non-zero X_i 's is binomially distributed.

Let
$$Y= \# \text{ of non-zero's}$$
.

$$P(X_i \neq 0) = 1 - P_L(X_i = 0) = 1 - e^{-\frac{1}{2}} = p \approx .095.6$$

$$Y \sim \text{Bin}(n_i p) \cdot \text{Want: } P(Y > \frac{1}{10})$$

$$= P\left(Y - pn > \left(\frac{1}{10} - p\right)n\right)$$

$$= P\left(\frac{Y - pn}{(np0p)} > \left(\frac{1}{10} - p\right)n\right)$$
by Central Limit
$$\Rightarrow P\left(\frac{Y - pn}{(np0p)} > \left(\frac{1}{10} - p\right)n\right)$$
Theorem
$$\Rightarrow P\left(\frac{(10 - p)}{p(1 - p)} + \frac{1}{10}\right) \left(\frac{(.005)}{(.095)(.905)} + \frac{1}{10}\right)$$

3. (20 points) 100 Emory students are asked their favorite course, the distribution of their answers is as follows:

Subject	ENG 317	TBT 102	MATH 362	PE 168	CHEM 350
Freq	10	15	50	20	5

Our initial impression of the popularities of the courses (in the order presented above) is $q_0 = .1, q_1 = .1, q_3 = .55, q_4 = .15, q_5 = .1$. We wish to test

$$H_0: q_0 = .1, \quad q_1 = .1, \quad q_3 = .56, \quad q_4 = .14, \quad q_5 = .1 \qquad vs \qquad \quad H_1: \text{all other possibilities}$$

Using the chi-square test, will we accept H_0 or H_1 at the 0.05 confidence level? At the 0.1 confidence level?

We compute:
$$\frac{\left(\frac{(x_{1}-q_{1}n)^{2}}{q_{1}n}\right)}{\left(\frac{(x_{1}-q_{1}n)^{2}}{q_{1}n}\right)} = \frac{\left(\frac{(x_{1}-10)^{2}}{10}\right)}{\left(\frac{(x_{1}-q_{1}n)^{2}}{10}\right)} + \frac{\left(\frac{50-56}{10}\right)^{2}}{\left(\frac{50-56}{10}\right)^{2}} + \frac{\left(\frac{20-14}{10}\right)^{2}}{\left(\frac{50-56}{10}\right)^{2}} + \frac{\left(\frac{50-56}{10}\right)^{2}}{\left(\frac{50-56}{10}\right)^{2}} + \frac{\left(\frac{20-14}{10}\right)^{2}}{\left(\frac{50-56}{10}\right)^{2}} + \frac{\left(\frac{50-56}{10}\right)^{2}}{\left(\frac{50-56}{10}\right)^{2}} + \frac{\left(\frac{20-14}{10}\right)^{2}}{\left(\frac{50-56}{10}\right)^{2}} + \frac{\left(\frac{50-56}{10}\right)^{2}}{\left(\frac{50-56}{10}\right)^{2}} + \frac{\left(\frac{50-56}{10}\right)^{2}}$$

4. (20 points) A sample of 20 $N(\mu, \sigma)$ random variables, where μ and σ are both unknown, is generated with $\bar{X} = 13.27$ and $S^2 = 24.15$.
a. (10 pts) Construct a 99%-confidence interval for μ . Note: Construct a real, not
approximate, confidence interval. We know I Land a dist w/ 19" degrees freedom.
S L+ El to so that P(-ta <t<ta)=,99,< td=""></t<ta)=,99,<>
10 look at our chart to tind to so P(1 <ta)=,995.< td=""></ta)=,995.<>
We take to = 2.861, and get interval
$(\bar{X} - 2.861 \frac{S}{m}, \bar{X} + 2.861 \frac{S}{m}) = (10.126, 16.414)$
as a 99% confidence into
b. (10 pts) Construct a 95% confidence interval for σ^2 . Hint: You know the distribution of $(n-1)S^2/\sigma^2$.
$\frac{(n-1)5^2}{6^2}$ $\chi^2(n-1) = \chi^2(19)$
Find a, 6 so Kat P(22(19)<6)=.975; take 6=32.850
Find a so that P(x2(19) <a)=.025; a="8.907</td" take=""></a)=.025;>
" $P(8.907 < \frac{(n-1)5^2}{6^2} < 32.852) = .95$
Rearranging We get:
Rearranging we get: $\frac{(n-1)5^2}{32.852} \neq \frac{(n-1)5^2}{8.907} = (13.967, 51.00515)$
is a 95% confidence interval for 6%.

5. (20 points) A sample X_1, \ldots, X_n of points in (0,1) is drawn from some pdf of the form $f(x;\theta) = \theta x^{\theta-1}$. We wish to test

$$H_0: \theta = 1; \quad vs \quad H_1: \theta > 1.$$

Design and explain a test to accomplish this goal, and find it's power function.

Hint: There are a *ton* of reasonable answers here. Two that come to mind: Base something on the mean, or on the distribution of the points - say the number of points in the first half of the strip.

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Practice 2 Solns

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\sum	5

1. (20 points)

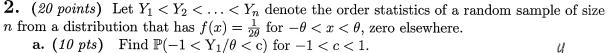
a. (10 pts) Let \bar{X} denote the mean of a random sample size n from a distribution that has mean μ and variance $\sigma^2 = 10$. Find n so that the probability is approximately 0.954 that the random interval $(\bar{X} - \frac{1}{2}, \bar{X} + \frac{1}{2})$ contains μ .

So:
$$M = 160$$

So: $M = 160$

Light 2 $M = 160$

b. (10 pts) Suppose \bar{X} is the mean of a random sample of size 25 from an $N(\mu, \sigma^2)$ distribution, where $S^2 = 10$. Find an exact 0.95 confidence interval for μ in terms of \bar{X} .



b. (10 pts) Find a 90% confidence interval for $-\theta$, if n = 5, and $Y_1 = -2.3$.

- 3. (20 points) Let $Y_1 < Y_2 < Y_3 < Y_4$ be the order statistics of a random sample of size n = 4

from a distribution with pdf
$$f(x;\theta) = \frac{2x}{\theta^2}$$
. The hypothesis $H_0:\theta=1$ is rejected and $H_1:\theta>1$ is accepted if the observed $Y_4 \geq c$.

a. (10 pts) Find the constant c so that the significance level is $\alpha=0.25$.

$$(Y_4) = 2 \sqrt{\frac{2}{2}} \sqrt{\frac{2}{2$$

Want:
$$\alpha = .25 = 1 - c^8$$

$$c^8 = .75$$

$$|c = .965|$$

b. (10 pts) Determine the power function of the test.

$$\frac{\chi}{c} = \frac{1}{10} \left(\frac{1}{14} > c \right) = \frac{6}{8} \frac{x^{2}}{08} dx$$

$$= |-\frac{c^8}{\Theta^8}$$

If
$$c = .965$$
,
= $\left| -\frac{.75}{\Theta^8} \right|$

4. (20 points) Emory students are asked how many A's they expect to receive this semester

It is proposed to fit this with a Poisson distribution.

Compute the corresponding chi-square test. How many degrees of freedom are associated with this test?

O= Dit know parameter: assume estimate it:

$$0 \approx X = \frac{24}{19}$$
, $9 = .2877$ $9 = .7877$ $9 = .7877$ $9 = .7877$ $9 = .7877$ $9 = .7877$ $9 = .7877$ $9 = .7877$ $9 = .7877$ $9 = .7877$ $9 = .7877$ $9 = .7877$ $9 = .7877$ $9 = .7877$ $9 = .7877$ $9 = .7877$ $9 = .7877$ $9 = .7877$ $9 = .7877$

Chi square computationi
$$(20-2685)^2 + (40-33.92)^2 + (25-21375)^2 + (12.73-10)^2$$

 $(25-21375)^2 + (12.73-10)^2$

Does the data result in the rejection of the Poisson model at the $\alpha = 0.05$ from

b. (10 pts) significance level?

We would not reject Voisson model, Since 4.04 < 5.991.

ble start With K-1=3 but like one since **5.** (20 points) X_1, \ldots, X_1 are independent $N(\mu, \sigma^2)$ random variables, where μ and σ are unknown.

a. (10 pts) Explain how to construct a 90% confidence interval for σ^2 using the fact that $(n-1)S^2/\sigma^2$ has the $\chi^2(n-1)$ distribution.

We know: $P(3.325 \le \chi^2 (n-1) \le 16.919) = .9$ $9 = P(3.325 \le \frac{(n-1)5^2}{6^2} \le 16.919)$ So: $(\frac{95^2}{16.919}, \frac{95^2}{3.325})$ is a 90% confidence interval for e^2

b. (10 pts) We wish to test $H_0: \sigma^2 = 2$ versus $H_1: \sigma^2 \neq 2$ using our interval at the 90% level. Would we accept or reject H_0 given the following data:

2.55 .95 -5.24 1.70 -2.17 1.43 -0.59 2.66 -2.59 -.94

Sorry for not giving you X, S here Didn't think of hw time consuming it was. Lesson learned "

X = .21 , S² = 6.564.

Get 90% confidence interval of:

(3.4916, 17.767). Since 2 ix outside this interval; we reject they and accept they