Math 362, Problem set 10

Due 4/25/10 (okay to turn in on 4/27 - I will be traveling, though)

1. (7.5.10) Let $X_1, \ldots, X_n$ be a random sample from a distribution of pdf $f(x;\theta) = \theta^2 x e^{-\theta x}$.
   (a) Argue that $Y = \sum X_i$ is a complete sufficient statistic for θ.
   (b) Compute $\mathbb{E}[1/Y]$ and find the function of Y which is the unique MVUE of θ.

2. (8.1.2) Let the random variable X have the pdf $f(x;\theta) = (1/\theta)e^{-x/\theta)}$ $0 < x < \theta$, zero elsewhere. Consider the simple hypothesis $H_0 : \theta = \theta' = 2$, and the alternative hypothesis $H_1 : \theta = \theta'' = 4$. Let X_1, X_2 denote a random sample of size 2 from this distribution. Show that the best test of H_0 against H_1 may be carried out by use of the statistic $X_1 + X_2$.

3. (8.2.8) Let $X_1, \ldots, X_n$ denote a random sample from a normal distribution $N(\theta, 16)$. Find the sample size n and a uniformly most powerful test of $H_0 : \theta = 25$ against $H_1 : \theta < 25$ with power function $\gamma(\theta)$ so that approximate $\gamma(25) = .10$ and $\gamma(23) = .9$.

4. (8.2.11) Let $X_1, \ldots, X_n$ be a random sample from a distribution with pdf $f(x;\theta) = \theta x^{\theta-1}$, $0 < x < 1$, zero elsewhere, where $\theta > 0$. Find a sufficient statsitic for θ and show that a uniformly most powerful test of $H_0 : \theta = 6$ against $H_1 : \theta < 6$ is based on this statistic.