# Math 362, Problem Set 6

Due 3/23/11

1. (6.1.2) Let $X_1, X_2, \ldots, X_n$ be a random sample from a $\Gamma(\alpha = 3, \beta = \theta)$ distribution, $0 < \theta < \infty$. Determine the mle of $\theta$.

2. (6.1.5) Suppose $X_1, \ldots, X_n$ are iid with pdf $f(x;\theta) = 2x/\theta^2$, $0 < x \leq \theta$, zero elsewhere. Find

   (a) The mle $\hat{\theta}$ for $\theta$.

   (b) The constant c so that $\mathbb{E}[c\hat{\theta}] = \theta$.

   (c) The mle for the median of the distribution.

3. (6.1.9) Suppose $X_1, \ldots, X_n$ are iid with pdf $f(x;\theta) = (1/\theta)e^{-x/\theta}$. Find the mle of $\mathbb{P}(X \leq 2)$.

4. (6.1.10) If $X_1, X_2, \ldots X_n$ be a random sample from a Bernoulli distribution with parameter $p$. If $p$ is restricted so that we know that $\frac{1}{2} \leq p \leq 1$, find the mle of this parameter.

5. (6.2.1) Prove that $\bar{X}$, the mean of a random sample of size $n$ from a distribution that is $N(\theta, \sigma^2)$ is, for every known $\sigma^2 > 0$, an efficient estimator of $\theta$.

6. (6.2.7') Let $X$ have a gamma distribution with $\alpha = 3$ and $\beta = \theta > 0$.

   (a) Find the Fisher information $I(\theta)$.

   (b) If $X_1, \ldots, X_n$ is a random sample from this distribution, show that the mle of $\theta$ is an efficient estimator of $\theta$¿

   (c) What is the asymptotic distribution of $\sqrt{n}(\hat{\theta} - \theta)$?

   **Note:** I changed $\alpha = 4$ in the original problem to $\alpha = 3$ since you computed the mle for $\theta$ in this case above.