

Math 362, Problem set 9

Due 4/20/10

1. (7.2.2) Prove that the sum of the observations of a random sample of size n from a Poisson distribution of having parameter θ , $0 < \theta < \infty$, is a sufficient statistic for θ .
2. (7.2.6) Let X_1, \dots, X_n be a random sample of size n from a beta distribution with parameters $\alpha = \theta$ and $\beta = 2$. Show that the product $X_1 X_2 \cdots X_n$ is a sufficient statistic for θ .
3. (7.2.8) What is the sufficient statistic for θ if the sample arises from a beta distribution in which $\alpha = \beta = \theta > 0$.
4. (7.3.3) If X_1, X_2 is a random sample of size 2 from a distribution having pdf $f(x; \theta) = (1/\theta)e^{-x/\theta}$, $0 < x < \infty$, zero elsewhere, find the joint pdf of the sufficient statistic $Y_1 = X_1 + X_2$ and $Y_2 = X_2$. Show that Y_2 is an unbiased estimator of θ with variance θ^2 . Find $\mathbb{E}[Y_2|y_1] = \varphi(y_1)$ and the variance of $\varphi(Y_1)$.
5. (7.3.4) Let $f(x, y) = (2/\theta^2)e^{-(x+y)/\theta}$, $0 < x < y < \infty$, zero elsewhere, be the joint pdf of the random variables X and Y .
 - Show that the mean and variance of Y are respectively $3\theta/2$ and $5\theta^2/4$.
 - Show that $\mathbb{E}[Y|x] = x + \theta$. In accordance with the theory we built up last semester, the expected value of $X + \theta$ is that of Y , namely, $3\theta/2$, and the variance of $X + \theta$ is less than that of Y . Show that the variance of $X + \theta$ is in fact $\theta^2/4$.
6. (7.4.6) Let a random sample of size n be taken from a distribution of the discrete type with pmf $f(x; \theta)$, $x = 1, 2, \dots, \theta$, zero elsewhere, where θ is an unknown positive integer.
 - Show that the largest observation, say Y , of the sample is a complete sufficient statistic for θ .
 - Prove that
$$\frac{Y^{n+1} - (Y-1)^{n+1}}{Y^n - (Y-1)^n}$$
is the unique MVUE for θ .