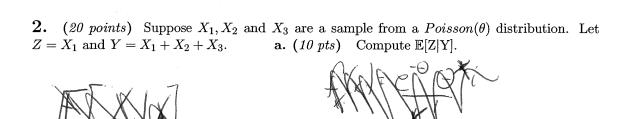
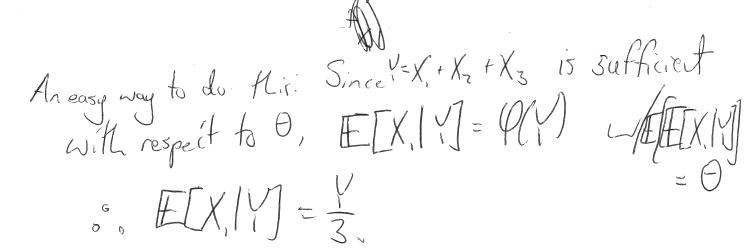
1. (20 points) Suppose $f(x;\theta) = e^{p(\theta)K(x)}$ for some functions p, K and X_1, \ldots, X_n are a sample for $f(x;\theta)$. Find a sufficient statistic for θ . Give a criteria on p and K for this to be complete.

 $\int_{\Omega} (x;\theta) = e^{\rho(\theta)} \sum_{i=1}^{n} K(x_i)^{n}$ 50 IK(X) is a complete exponential IF p(0)\$, K(2)\$0, and the Att. range dues not depend on 0, this is regular, exponential and so IK(x) is complete.

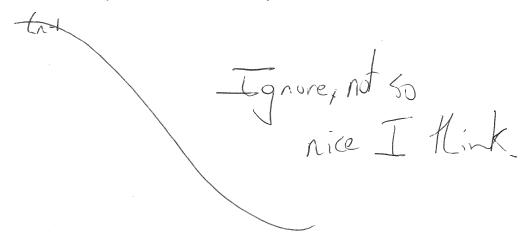




b. (10 pts) Compute $\mathbb{E}[Z]$ and $\mathbb{E}[Z|Y]$ along with Var(Z) and $Var(\mathbb{E}[Z|Y])$.

3. (20 points) a. (10 pts) Suppose X_1, \ldots, X_n have the $N(\mu, \theta)$ distribution, where μ is unknown. Let $S^2 = \frac{1}{n-1} \sum (X_i - \bar{X})^2$. Find the most powerful test of $H_0: \theta = 1$ versus $H_1: \theta = 2$.

Hint: Recall that you know the distribution of S^2 .



b. (10 pts) Suppose X_1, \ldots, X_n have the $N(\theta, 1)$ distribution. Is there a uniformly most powerful test of $H_0: \theta = \emptyset$ versus $H_1: \theta \neq 1$.

M:
$$L(\Phi) = \exp(-\frac{1}{2}(\Theta'-1)ZX + \frac{1}{2}n\Theta'-\frac{1}{2}n)$$

This yields a test of Accept H, if $Z_1X_1 \ge c$ if $\Theta' \ge 1$

and Accept H, if $Z_1X_2 \le c$ if $\Theta' \ge 1$,

Since tests are diff. no uniformly most powerful test.

4. (20 points) Consider the two loss functions $L_1(x,\theta) = (x-\theta)^2$ and $L_2(x,\theta) = |x-\theta|$ and their associated risk functions $R_1(x,\theta)$ and $R_2(x,\theta)$. For a random variable X, which is larger $R_1(x,\theta)$ or $R_2(x,\theta)^2$ and why?

 $E[X^2] \ge E[X]^2$ by Jensen and Horefore $R_1(x,\theta) \ge R_2(x,\theta)^2$