1. (*20 points*) Suppose $f(x;\theta) = e^{p(\theta)K(x)}$ for some functions p, K and $X_1, \ldots, X_n$ are a sample for $f(x;\theta)$. Find a sufficient statistic for θ. Give a criteria on p and K for this to be complete.

~~If~~ $f_{X_1 \cdots X_n}(x;\theta) = e^{p(\theta)\sum K(x)}$

so $\sum K(x)$ is a complete exponential family.

If $p(\theta) \neq 0$, $K(x) \neq 0$, and the ~~[illegible]~~ range does not depend on θ, this is regular exponential and so $\sum K(x)$ is complete.

2. (*20 points*) Suppose X_1, X_2 and X_3 are a sample from a *Poisson*(θ) distribution. Let $Z = X_1$ and $Y = X_1 + X_2 + X_3$. **a.** (*10 pts*) Compute $\mathbb{E}[Z|Y]$.

An easy way to do this: Since $Y = X_1 + X_2 + X_3$ is sufficient with respect to θ, $\mathbb{E}[X_1|Y] = \varphi(Y)$ w/ $\mathbb{E}[\mathbb{E}[X_1|Y]] = \theta$

$\therefore \mathbb{E}[X_1|Y] = \frac{Y}{3}$.

b. (*10 pts*) Compute $\mathbb{E}[Z]$ and $\mathbb{E}[Z|Y]$ along with $\mathrm{Var}(Z)$ and $\mathrm{Var}(\mathbb{E}[Z|Y])$.

$\mathbb{E}[Z] = \mathbb{E}[\mathbb{E}[Z|Y]] = \theta$

$\mathrm{Var}(Z) = \theta$ $\quad$ $\mathrm{Var}(\mathbb{E}[Z|Y]) = \frac{\theta}{3}$.

3. (*20 points*) **a.** (*10 pts*) Suppose $X_1, \ldots, X_n$ have the $N(\mu, \theta)$ distribution, where μ is unknown. Let $S^2 = \frac{1}{n-1}\sum(X_i - \bar{X})^2$. Find the most powerful test of $H_0 : \theta = 1$ versus $H_1 : \theta = 2$.
Hint: Recall that you know the distribution of S^2.

~~t_{n-1}~~

~~Ignore, not so nice I think~~

b. (*10 pts*) Suppose $X_1, \ldots, X_n$ have the $N(\theta, 1)$ distribution. Is there a uniformly most powerful test of $H_0 : \theta = 1$ versus $H_1 : \theta \neq 1$.

No:

$$\frac{L(1)}{L(\theta')} = \exp\left(-\frac{1}{2}(\theta'-1)\sum X_i + \frac{1}{2}n\theta'^2 - \frac{1}{2}n\right)$$

This yields a test of
Accept H_1 if $\sum X_i \geq c$ if $\theta' > 1$

and Accept H_1 if $\sum X_i \leq c$ if $\theta' < 1$,
Since tests are diff. no uniformly most powerful test.

4. (*20 points*) Consider the two loss functions $L_1(x, \theta) = (x - \theta)^2$ and $L_2(x, \theta) = |x - \theta|$ and their associated risk functions $R_1(x, \theta)$ and $R_2(x, \theta)$. For a random variable X, which is larger $R_1(x, \theta)$ or $R_2(x, \theta)^2$ and why?

$\mathbb{E}[X^2] \geq \mathbb{E}[X]^2$ by Jensen

and therefore

$R_1(x, \theta) \geq R_2(x, \theta)^2$