Mile High Conference on Quasigroups, Loops, and Nonassociative Systems

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Abstracts

Local algebras of a differentiable quasigroup

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We study local differentiable quasigroups and their local algebras defined by the second and third orders terms of Taylor's decomposition of a function defining an operation in local loops. In local algebras, we define the commutators and associators connected by a third-degree relation generalizing the Jacobi identity in Lie algebras. Hofmann and Strambach named the local algebras mentioned above Akivis algebras.

In general, an Akivis algebra does not uniquely determine a differentiable quasigroup, but for Moufang and Bol quasigroup, it enjoys this property.

We consider also prolonged Akivis algebras and prove that a local algebra defined in a fourth-order neighborhood uniquely determines a monoassociative quasigroup.

The last two results give a generalization of the classical converse third Lie theorem on determination of a local Lie group by its Lie algebra.

As an illustration, we consider local differentiable quasigroups defined on the Grassmannian $\mathbb{G}(1, r+1)$ by a triple of hypersurfaces in the projective space $\mathbb{P}^{r+1}$.

On the existence of irreducible n-quasigroups (Solution of Belousov's problem)

M. A. Akivis (Jerusalem Institute of Technology, Jerusalem, Israel)
V. V. Goldberg (New Jersey Institute of Technology, Newark, NJ, USA)*

The authors prove that a local n-quasigroup defined by the equation

$$x_{n+1} = F(x_1, \dots, x_n) = \frac{f_1(x_1) + \dots + f_n(x_n)}{x_1 + \dots + x_n},$$

where $f_i(x_i)$, $i, j = 1, \dots, n$, are arbitrary functions, is irreducible if and only if any two functions $f_i(x_i)$ and $f_j(x_j)$, $i \neq j$, are not both linear homogeneous, or these functions are linear homogeneous but $\frac{f_i(x_i)}{x_i} \neq \frac{f_j(x_j)}{x_j}$.

This gives a solution of Belousov's problem to construct examples of irreducible n-quasigroups for any $n \geq 3$.

Finite loops and projective planes

Michael Aschbacher (California Institute of Technology, Pasadena, CA, USA)

Projective planes are coordinatized by ternary rings (R, T) with an associated "ring" $R = (R, +, \cdot)$, such that $(R, +)$ is a loop with identity 0, and $(R^{\#}, \cdot)$ is a loop with identity 1, where $R^{\#} = R - \{0\}$. We consider finite projective planes linearly coordinatized (ie. $T(x, y, z) = xy + z$) by a right distributive (ie. $(x + y)z = xz + yz$) ring R. Most known finite planes are of this sort. We use techniques from finite group theory and loop theory to study such planes and their associated rings and loops. Various partial results suggest that possibly either $(R, +)$ or $(R^{\#}, \cdot)$ is a group. Much of the analysis involves the following interesting class of loops: Loops $(R, +)$ admitting a group of automorphisms transitive on $R^{\#}$.

Bol loops that are centrally nilpotent of class two

Orin Chein (Temple University, Philadelphia, PA, USA)

After considering the question Why nilpotence class 2? from both historical and practical perspectives, I will investigate identities that hold in (right) Bol loops with this property and use these to create some new constructions of such loops.

Abelian inner mappings and nilpotency class greater than two

Piroska Csörgő (Eotvos University, Budapest, Hungary)

By T. Kepka and M. Niemenmaa if the inner mapping group of a finite loop Q is abelian, then the loop Q is centrally nilpotent. For a long time there was no example of a nilpotency degree greater than two. In the nineties T. Kepka raised the following problem: whether every finite loop with abelian inner mapping group is centrally nilpotent of class at most two. For many years the prevailing opinion has been that all such loops have to be of nilpotency degree two. The converse is always true by Bruck, i.e. the nilpotency class two of the loop Q implies the inner mapping group $I(Q)$ is abelian.

After describing the problem in terms of transversals I tried to characterize by means of group theory the least counterexample. I expected to find enough properties of the counterexample that would refute its existence. By using these results, supposing special properties, I choose some parameters and finally I constructed a conterexample loop Q of order 2^7, such that the multiplication group $M(Q)$ is of order 2^{13}, the inner mapping group $I(Q)$ is elementary abelian of order 2^6, for the normal closure M_0 of $I(Q)$ in $M(Q)$, M_0 is of order 2^{10} and the factor group $M(Q)/M_0$ is elementary abelian of order 2^3, furthermore the nilpotency class of this loop Q is greater than two.

The self-distributive structure of parenthesized braids

Patrick Dehornoy (Laboratoire de Mathematiques Nicolas Oresme, Universite de Caen, Caen, France)

Artin's braid group B_∞ is equipped with a remarkable left self-distributive operation that reflects a deep connection between braids and the self-distributive law. A similar connection exists between R. Thompson's group F and the associativity law. Mixing the two laws leads to introducing a new group $B_\bullet$ that extends both B_∞ and F, and whose elements can be viewed as braids in which the distances between the strands need not be uniform. Many properties of B_∞ extend to $B_\bullet$, in particular the connection with the fundamental group of a punctured surface, the embeddability in the automorphisms of a free group, and the existence of a self-distributive structure. Now the most interesting algebraic point is that $B_\bullet$ comes equipped with a second operation compatible with the self-distributive operation in a natural way ("augmented LD-system"), which cannot be the case for ordinary braids.

Conjugacy closedness and related matters

Aleš Drápal, Charles University, Prague, Czech Republic

Every conjugacy closed loop $Q(*)$ with $A(Q) \leq Z(Q)$ has a subloop S such that Q/S is abelian of exponent two, and $S(*)$ can be obtained from a group $G = S(\cdot)$ and a symmetric quadri-additive mapping $b : G \times G \to G$ so that $x * y = x \cdot y \cdot b(x, y)$.

What means quadri-additive? Let G and H be abelian groups. A mapping $b : G \times G \to H$ is said to be *quadri-additive* if

1. $b(\lambda x, \mu y) = \lambda^2 \mu b(x, y)$ for $\lambda, \mu \in \mathbb{Z}$;
2. $b(x, y + z) = b(x, y) + b(x, z)$; and
3. $(x, y) \mapsto b(x + y, z) - b(x, z) - b(y, z)$ is a biadditive mapping $G \times G \to H$ for every $z \in G$.

For nonabelian groups, we say that $b : G \times G \to H$ is quadri-additive if b induces a quadri-additive mapping $G/G' \times G/G' \to Z(H)$. (Other results will be discussed, too.)

Loops over nearfields

Clifton E. Ealy, Jr. (Western Michigan University, Kalamazoo, MI, USA)

$(N, +, \circ)$ is a *nearfield* if $(N, +)$ is an abelian group with identity 0, $(N \setminus \{0\}, \circ)$ is a multiplicative group with identity 1 not equal to 0, and there is only one distributive law. In this talk, I will define matrix loops over nearfields and consider some of their properties.

Description of the subalgebras of Zorn's algebra over F_q, $q = 2, 3$

Maria de Lourdes Merlini Giuliani (Universidade Federal de Santa Maria, Santa Maria, Brazil)

In this talk I present the classification of all subalgebras of Zorn's algebra $Z(q)$ over F_q, where $q = 2, 3$; and I present the number of subalgebras for each case. I show that the isomorphic subalgebras are conjugate in respect to the automorphism group of $Z(q)$. Furthermore, as a consequence I obtain that the maximal subloops of the corresponding simple Moufang loops $M(q)$ are conjugate in the automorphism groups of $Z(q)$.

Eigenvalues of generic adjoint maps in comtrans algebras of bilinear spaces

Bokhee Im (Chonnam National University, Kwangju, Korea)*
Jonathan D. H. Smith (Iowa State University, Ames, IA, USA)

Comtrans algebras are unital modules over a commutative ring R, equipped with two basic trilinear operations: a *commutator* $[x, y, z]$ satisfying the *left alternative identity*

$$[x, x, y] = 0,$$

and a *translator* $\langle x, y, z\rangle$ satisfying the *Jacobi identity*

$$\langle x, y, z\rangle + \langle y, z, x\rangle + \langle z, x, y\rangle = 0,$$

such that together the commutator and translator satisfy the *comtrans identity*

$$[x, y, x] = \langle x, y, x\rangle.$$

A long-term goal of the research effort devoted to comtrans algebras is to develop a general structure theory. As a first step towards this goal, we focus on the eigenvalues of generic adjoint maps of comtrans algebras $CT(E, \beta)$ of bilinear spaces (E, β), and the extent to which knowledge of these eigenvalues and their multiplicities serves to specify the algebras up to isomorphism within certain classes. Such a comtrans algebra $CT(E, \beta)$ has underlying module E and its algebra structure is defined by

$$[x, y, z] = y\beta(x, z) - x\beta(y, z)$$

and

$$\langle x, y, z\rangle = y\beta(z, x) - x\beta(y, z).$$

The construction of loops by varying group tables

Kenneth Johnson (Penn State Abington, Abington, PA, USA)

The object of this talk is a discussion of the ways in which loops can be constructed from groups by using combinatorial ideas and symmetry. The starting point is the observation by the presenter and P. Vojtechovsky that the multiplication tables of groups with respect to right division are much more symmetrical than the ordinary tables, in that they can be written as block circulants with further symmetry properties. A weakening of the strong symmetry of these tables produces tables corresponding to loops. For example, there is a nice construction for the Moufang loop of order 12 (using the right division operation) as

1	3	2	4	5	6	7	8	9	10	11	12
2	1	3	6	4	5	9	7	8	12	10	11
3	2	1	5	6	4	8	9	7	11	12	10
4	6	5	1	2	3	10	12	11	7	9	8
5	4	6	3	1	2	12	11	10	9	8	7
6	5	4	2	3	1	11	10	12	8	7	9
7	9	8	10	12	11	1	2	3	4	6	5
8	7	9	12	11	10	3	1	2	6	5	4
9	8	7	11	10	12	2	3	1	5	4	6
10	12	11	7	9	8	4	6	5	1	2	3
11	10	12	9	8	7	6	5	4	3	1	2
12	11	10	8	7	9	5	4	6	2	3	1

This can be written more briefly as

$$\begin{array}{cccc}
C(1,3,2) & C(4,5,6) & C(7,8,9) & C(10,11,12) \\
C(4,6,5) & C(1,2,3) & RC(10,12,11) & RC(7,9,8) \\
C(7,9,8) & RC(10,12,11) & C(1,2,3) & RC(4,6,5) \\
C(10,12,11) & RC(7,9,8) & RC(4,6,5) & C(1,2,3)
\end{array}$$

where $C(i, j, k)$ denotes a circulant and $RC(i, j, k)$ a reverse circulant i.e. each row is obtained from the previous by a left shift. Diassociativity more or less fixes all the blocks except for those in the $(2, 3)$, $(2, 4)$ and $(3, 4)$ positions, together with their reflections in the diagonal, and in these positions the reverse circulants appear. The loops from the Chein construction based on dihedral groups can be presented in an analogous way. It is an interesting question as to whether the Moufang condition may be verified by showing the closure of the corresponding diagrams in web geometry, which of course can be translated into the conditions on the tables.

Although it is unlikely that simple loops of an interesting nature may be produced by some variations of symmetry as above, (for example it does not seem possible to construct the simple Moufang loop of order 120 in a similar way to that above) this naive approach can produce loops which automatically satisfy certain conditions, for example having a composition series of a certain type. Moreover, by varying tables coming from groups we can produce tables which automatically correspond to non-associative loops and which have character tables which are specified. The following example illustrates. The table (under right multiplication) for any dihedral group of order 2n may be written

$$\begin{array}{cc} C(1,n,\ldots,2) & C(n+1,n+2,\ldots,2n) \\ C(n+1,2n,\ldots,n+2) & C(1,2,\ldots,n) \end{array}$$

If σ is any non-identity permutation on $\{2,\ldots,n\}$ the table

$$\begin{array}{cc} C(1,n,\ldots,2) & C(n+1,n+2,\ldots,2n) \\ C(n+1,2n,\ldots,n+2) & C(1,\sigma(2),\ldots,\sigma(n)) \end{array}$$

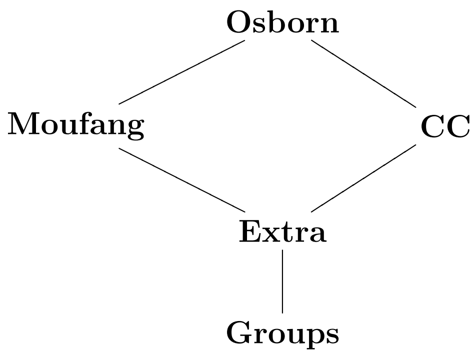
corresponds to a non-associative loop. Question: how many distinct isomorphism classes of loops can be produced in this way and what identities do they satisfy?

If time permits I will also indicate other symmetrical constructions which produce families of loops.

A survey of Osborn loops

Michael K. Kinyon (Indiana University South Bend, South Bend, IN, USA)

The study of Moufang loops is a time-honored tradition within loop theory, and connects to many different areas of mathematics. The study of CC-loops has taken off in the last few years because of the dissemination of Basarab's theorem (the quotient of a CC-loop by its nucleus is an abelian group). This talk will mostly consist of a mini-course in a common generalization of the two varieties, namely the variety of *Osborn loops*. The picture to have in mind is the following, although I will show a much more elaborate one in the talk.



The definition of Osborn loops is quite natural, and a surprising amount of the basic structure of both Moufang and CC-loops is already present in Osborn loops. I will survey published results (mostly due to Basarab) and some new ones as well. Interestingly, one can even generalize some of the basic results in the theory of commutative Moufang loops to a certain class of Osborn loops.

I will also address those in the audience who prefer quasigroups to loops. There is a conjectural relationship between Osborn loops and the (as yet) unstudied class of *conjugacy closed quasigroups*, that is, quasigroups in which the sets of left and right translations are each closed under self-conjugation.

If there is time (though I doubt there will be), I will briefly discuss the one-sided case, namely common generalizations of Bol loops and RCC-loops.

Linear balanced quasigroup identities

Aleksandar Krapež (Matematički institut SANU, Beograd, Serbia and Montenegro)

We consider quasigroup identities which contain only the multiplication symbol $\cdot$. Such an identity is *linear* if every variable appears at most once in $s(t)$ and is *balanced* if the set $var(s)$ of variables of s is equal to $var(t)$.

Every linear balanced quasigroup identity is either Belousov (i.e. a consequence of commutativity) or non–Belousov (i.e. implying group isotopy). A. Krapež and M. A. Taylor (Czechoslovak Math. J. 43 (118), (1993)) proved that every set of Belousov identities is equivalent to a single *normal* Belousov identity. Consequently the lattice of nontrivial Belousov varieties is isomorphic to the lattice of odd numbers under divisibility.

Non–Belousov identities are more difficult to handle. W. Förg–Rob and A. Krapež defined *height–preserving* identities ($s = t$ is height–preserving if every variable x is of equal height in s and t) and proved that a quasigroup satisfying such an identity is a T–quasigroup in which $branch(x, s) = branch(x, t)$ for all $x \in var(s)$. Height–preserving identities can be understood as permutations transforming the tree of s to that of t. Belousov identities then correspond to isomorphisms of trees of s and t.

Varieties of quasigroups satisfying height–preserving identities of height up to h can be related to special subgroups of the group S_{2^h}.

Reflection quasigroups: Algebraic models for symmetric spaces with midpoints

Jimmie Lawson (Louisiana State University, Baton Rouge, LA, USA)*
Yongdo Lim (Kyungpook National University, Taegu, Korea)

A reflection quasigroup $(Q, \cdot)$ is an idempotent quasigroup for which each left translation is an involutive automorphism. We give a geometric interpretation to such structures by viewing $p \cdot x = y$ to mean that the point reflection through p carries x to y. (The operation of point reflection in a vector space, $p \cdot x := 2p - x$, $2 \neq 0$, is a basic example.) From this perspective the unique solution $x = b/a$ of $x \cdot a = b$ yields the unique "midpoint" between a and b; this midpoint plays a crucial role in the theory. These structures turn out to have characterizations in terms of, or even categorical equivalences with, a variety of other algebraic structures: uniquely 2-divisible twisted subgroups, transversal twisted subgroups of involutive groups, a special class of loops called B-loops, and uniquely 2-divisible gyrocommutative gyrogroups.

We consider reflection quasigroups arising from a general class of symmetric spaces called lineated symmetric spaces. Our primary interest is the case that these symmetric spaces are (differentiable) Banach manifolds, in which case they exhibit an interesting geometric structure, and particularly in the metric case, where it is assumed the symmetric space carries a convex metric, an invariant complete metric contracting the square root function. One major result is that the distance function between points evolving over time on two geodesics is a convex function. Primary examples arise from involutive Banach-Lie groups (G, σ) admitting a polar decomposition $G = P \cdot K$, where K is the subgroup fixed by

σ and P is the associated symmetric space. We consider an appropriate notion of seminegative curvature for such symmetric spaces endowed with an invariant Finsler metric and prove that the corresponding length metric must be a convex metric. The preceding results provide a general framework for the interesting Finsler geometry of the space of positive Hermitian elements of a C^*-algebra that has emerged in recent years.

Release of Prover9

William McCune (Mathematics and Computer Science Division, Argonne National Laboratory, IL, USA)

This talk marks the release of Prover9, a new theorem prover for first-order and equational logic. Prover9 has a fully automatic mode, but difficult problems frequently require some human guidance, which will be a focus of the talk. Mace4, a program that searches for finite counterexamples, will also be covered. Example problems in nonassociative algebras will be presented, and the conference participants will be invited and encouraged to challenge Prover9 and Mace4 with new problems. At the time of the conference, Prover9 will be available from http://www.mcs.anl.gov/∼mccune/prover9/.

Planes, nets and webs

G. Eric Moorhouse (University of Wyoming, Laramie, WY, USA)

The main open questions in the study of finite projective planes concern the possible orders of finite planes, and the question of whether planes of prime order are necessarily the classical ones. A promising approach to both questions relies on conjectured bounds for ranks of finite nets (i.e. rank of the incidence matrix over a field of positive characteristic). The conjectured rank bounds for (finite) nets agree with the known rank bounds for (infinite) webs, but different mathematical tools are required in the finite case.

In 1991 I verified the conjectured rank bounds for 3-nets (those having 3 parallel classes of lines) using loop theory. I will describe recent progress in the case of 4-nets, using the method of exponential sums.

Quasigroups, bigroups and local bigroups

Yuri M. Movsisyan (Yerevan State University, Yerevan, Armenia)

In this talk the concepts of bigroups and local bigroups will be discussed, and quasigroups in local bigroups of operations will be characterized. The obtained results are applied in Steiner, Stein and Belousov quasigroups.

On nilpotent Moufang loops

Gábor P. Nagy (University of Szeged, Bolyai Institute, Szeged, Hungary)

Despite the importance of Moufang loops in the theory of quasigroups and loops, there are astonishingly few general constructions for nilpotent Moufang loops. The only exceptions are the classes of commutative Moufang loops and Moufang 2-loops (due to Chein's construction and its generalizations). Together with M. Valsecchi, we proved the following.

Theorem 1 *Let M be a group and k a positive integer. Let $f : M \to Z(M)$ and $g : M \times M \to Z(M)$ be maps with the following properties:*

1. *f and g vanish on $f(M) \cup g(M, M)$, that is,*

$$f(f(m)) = f(g(m_1, m_2)) = g(f(m_1), m_2) = g(g(m_1, m_2), m_3) = 1$$

for each $m, m_1, m_2, m_3 \in M$.

2. *g is bilinear and alternating, which means*

$$\begin{aligned} g(m_1m_2, m_3) &= g(m_1, m_3)g(m_2, m_3), \\ g(m_1, m_2) &= g(m_2, m_1)^{-1}, \\ g(m, m) &= 1 \end{aligned}$$

for each $m, m_1, m_2, m_3 \in M$.

3. *f satisfies*

$$f(m_1m_2) = f(m_1)f(m_2)g(m_1, m_2)^3 \quad (1)$$

for all $m_1, m_2 \in M$ and $f(m)^k = 1$ for all $m \in M$.

Define the operation

$$(i_1, m_1) \cdot (i_2, m_2) = (i_1 + i_2, m_1m_2f(m_1)^{i_2}g(m_1, m_2)^{i_1+2i_2}) \quad (2)$$

on the set $L = \mathbf{Z}_k \times M$. Then, $(L, \cdot)$ is a Moufang loop with unit $(0, 1)$.

It turns out that these loops can be very effectively used in the following areas:
1) Classification of Moufang loops of order p^5, $p > 3$.
2) Structural properties and examples of Moufang loops with central associators.
3) Minimally nonassociative Moufang p-loops.

Powers and alternative laws

Nicholas Ormes (University of Denver, Denver, CO, USA)

A groupoid is alternative if it satisfies the alternative laws: $x(xy) = (xx)y$ and $x(yy) = (xy)y$. Let A be the free alternative groupoid with generator x. An nth power of x is an element of A which is obtained by multiplying x with itself n times. Of course, nth powers of x need not be unique since there are many ways to parenthesize the multiplication. Each application of an alternative law passes from one nth power of x to another and a basic question is - for which values of n are nth powers of x unique? We show that the answer is only when $n \leq 5$.

The technique used is to study the action of multiplication by 2 modulo n. In dynamical terms, this is a quotient of the original action of application of an alternative law (or the inverse of such an application), but is much simpler. In particular, if we show that this map has orbits which are not complete, we show that powers cannot be unique. We also use this action to say something about the problem of finding alternative loops without two-sided inverses.

Operads and derivations

Eugen Paal (Tallinn University of Technology, Tallinn, Estonia)*
Peeter Puusemp (Tallinn University of Technology, Tallinn, Estonia)

(See the electronic abstract posted online.)

On C-loops

J. D. Phillips (Wabash College, Crawfordsville, IN, USA)

There are relatively few papers in the literature about C-loops in spite of the fact that they are loops of so-called Bol-Moufang type and have very rich structure. This talk outlines some of the basic structural features of C-loops and shows where they fit into the larger hierarchy of loops of Bol-Moufang type.

Greedy quasigroups

T. A. Rice (Iowa State University, Ames, IA, USA)

A *greedy quasigroup* Q_s, is a quasigroup structure on the natural numbers generated by fixing $0 \cdot 0 = s$. The number s is called the *seed*. The rest of the multiplication table is filled in using a greedy algorithm. For example, Q_0 is an elementary 2-group. I will discuss algebraic properties, including whether the Q_s are isomorphic to each other, and properties of the multiplication groups. Finally I will briefly discuss applications to the theory of combinatorial games.

Varieties of binary modes

Anna Romanowska (Warsaw University of Technology, Warsaw, Poland)

A *binary mode* or *groupoid mode* is a set with a binary operation that is idempotent and entropic. An identity is *regular* if the same sets of variables appear on each side. A variety is *irregular* if it is specified by at least one irregular identity. The *regularization* of a variety is the class of models of the regular identities satisfied by each member of the variety.

We describe a lower part of the lattice of varieties of binary modes. The lower part of the lattice splits naturally into two subparts. One consists of irregular varieties. The other consists of their regularizations. The regularized varieties are easily described. The irregular varieties split further into two parts. One consists of idempotent and entropic quasigroups. The other consists of varieties of so-called *reductive groupoid modes* (including for example the variety of differential groupoids).

On the theory of smooth M-loops

Liudmila Sabinina (Facultad de Ciencias, UAEM, Cuernavaca, Mexico)

The smooth CC-loops are considered in the context of the theory of smooth M-loops. We would like to discuss some properties of these loops.

A loop $< Q, \cdot, \backslash, /, e >$ with the identity

$$x \cdot (y \cdot z) = (x \cdot (y \cdot \phi(x))(J\phi(x) \cdot z)$$

where $\phi(x) : Q \to Q$, $J : Q \to Q$ are maps, and $x \cdot Jx = e$, is called an M-loop.

Right versus Left: Relativistic dynamics in the Einstein velocity loop

Tzvi Scarr (Jerusalem College of Technology, Jerusalem, Israel)

We explore the loop $(D, \oplus_E)$ of Einstein velocity addition on the ball $D = \{v \in \mathbb{R}^3 : |v| \leq c\}$ of relativistically admissible velocities. This loop, which is also a gyrocommutative gyrogroup, can be used to derive the relativistic dynamics equation that describes, for example, the evolution of the velocity of a charged particle in an electromagnetic field.

The natural first step in developing dynamics from the loop $(D, \oplus_E)$ is to look at the *translations* of the loop. However, since the addition $\oplus_E$ is, in general, not commutative (in fact, it is commutative only for parallel velocities), one must decide whether to use the *right* or the *left* translations. There seems to be no *a priori* preference for either one. Indeed, one can ask whether it makes a difference. Is the dynamics which stems from the left translations different from the dynamics of the right translations? If so, which dynamics does nature choose?

We will show how the *left* translations $\varphi_a : D \to D$, $\varphi_a(v) = a \oplus_E v$ lead to the usual relativistic dynamics equation. The development here is straightforward, due mainly to two facts:

- the left translations of $(D, \oplus_E)$ are *projective* automorphisms of D
- the inverse of the left translation φ_a is again a left translation, namely φ_{-a}.

In contrast, the *right* translations $\phi_a : D \to D$, $\phi_a(v) = v \oplus_E a$ are problematic. They are not projective maps. They're not even analytic! And the inverse of a right translation is not a right translation. A closer look at the *physical meaning* of right and left translations reveals that there is an inherent difference between them. We will explain this asymmetry and discuss possible directions for developing the "right" dynamics.

On identities of isotopy closure of variety of groups

Khalil Shahbazpour (Urmia University, Urmia, Iran)

In this talk we will dicuss the following result.

Theorem. *A quasigroup $G(\cdot)$ is an isotope of group if and only if one of the following identities holds for $G(\cdot)$.*

(a) $x\{z\backslash[(z/u)v]\} = \{[x(z\backslash z)]/u\}v$

(b) $x\{u\backslash[(z/u)v]\} = \{[x(u\backslash z)]/u\}v$

(c) $x\{z\backslash[(u/u)v]\} = \{[x(z\backslash u)]/u\}v$

(d) $x[y\backslash\{[(yy)/z]u\}] = [\{x[y\backslash(yy)]\}/z]u$

(e) $x[y\backslash\{[(yz)/y]u\}] = [\{x[y\backslash(yz)]\}/y]u$

$(f)\quad x[z\backslash\{[(yy)/y]u\}] = [\{x[z\backslash(yy)]\}/y]u$

References

[1] Movsisyan Yu. M., *Introduction to the Theory of Algebras with Hyperidentities*, Yerevan State University Press, Yerevan, 1986.

Loop-based identities in the double of a central pique

Jonathan D. H. Smith (Iowa State University, Ames, IA, USA)

A *pique* is a quasigroup with a pointed idempotent. A central pique is a principal isotope of an abelian group, its corresponding loop or *cloop*, the components of the isotopy being automorphisms of the cloop. The characters of a central pique's cloop form a dual pique. The conjugacy classes of the dual correspond to the characters of the primal; indeed the unitary character table of the dual is the inverse of the unitary character table of the primal. Together with its dual, a central pique forms a structure known as the *double*.

In what may well be the first application of one part of quasigroup theory to another, it is shown that the double of a central pique satisfies identities indexed by loops of 2-power order. These identities project onto the unit circle to yield identities involving character values.

Introduction to Evolution Algebra

Jianjun Paul Tian (The Ohio State University, Columbus, OH, USA)

Behind the phenomena of genetics and stochastic processes, we find there is an intrinsic algebraic structure. We call it — evolution algebra. Evolution algebras are non-associative, non-power-associative Banach algebras and have many connections with other mathematical fields including graph theory, group theory, Markov chains, dynamic systems, knot theory, 3-manifolds and the study of the Riemann-zeta function. In the present talk, we will introduce the basic concepts of evolution algebras and hierarchical structure theory. One of the unusual features of of an evolution algebra is that it possesses an evolution operator. This evolution operator reveals the dynamic information of an evolution algebra. What makes the theory of evolution algebras different from the classical theory of algebras is that in evolution algebras, we can have two different types of generators: algebraically persistent generators and algebraically transient generators. The basic notions of algebraic persistency and algebraic transiency, and their relative versions, lead to a hierarchical structure on an evolution algebra. Dynamically, this hierarchical structure displays the direction of the flow induced by the evolution operator. Algebraically, this hierarchical structure is given in the form of a sequence of semi-direct-sum decompositions of a general evolution algebra. The dynamic nature of this hierarchical structure is what makes the notion of an evolution algebra applicable to the study of stochastic processes and many other objects in different fields.

Code loops of both parities

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Even code loops introduced by Griess are well known. Less known are odd code loops defined by Richardson. We show that Richardson's definition can be generalized so that the code loops of both parities can be characterized in a uniform way. We therefore argue that the generalized definition is more natural. This is joint work with Aleš Drápal.

When is an n-ary quasigroup an iterated group isotope?

Thomas Zaslavsky (Binghamton University (SUNY), Binghamton, NY, USA)

Rumor has it that Belousov conjectured that an n-ary quasigroup is isotopic to an iterated group if its factorization graph is 3-connected. I have proved this conjecture by a new method employing a kind of branched covering of the factorization graph.

Belousov *et al.* reportedly also proved that an n-ary quasigroup (Q, f), with $n > 2$, is an iterated group isotope if $|Q| \leq 3$; but for $|Q| = 3$ the proof was too long to publish. (I have not been able to find a published proof.) I have a short proof based on the concept of a residual quasigroup of f, that is, a k-ary quasigroup obtained by fixing $n - k$ independent variables in f. If every residual ternary quasigroup is isotopic to an iterated group, then f is isotopic to an iterated group (but that conclusion does not follow if every residual binary quasigroup is an iterated group isotope).