

Local algebras of a differentiable quasigroup

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We study local differentiable quasigroups and their local algebras defined by the second and third orders terms of Taylor's decomposition of a function defining an operation in local loops. In local algebras, we define the commutators and associators connected by a third-degree relation generalizing the Jacobi identity in Lie algebras. Hofmann and Strambach named the local algebras mentioned above Akivis algebras.

In general, an Akivis algebra does not uniquely determine a differentiable quasigroup, but for Moufang and Bol quasigroup, it enjoys this property.

We consider also prolonged Akivis algebras and prove that a local algebra defined in a fourth-order neighborhood uniquely determines a monoassociative quasigroup.

The last two results give a generalization of the classical converse third Lie theorem on determination of a local Lie group by its Lie algebra.

As an illustration, we consider local differentiable quasigroups defined on the Grassmannian $\mathbb{G}(1, r + 1)$ by a triple of hypersurfaces in the projective space \mathbb{P}^{r+1} .