On the existence of irreducible n-quasigroups (Solution of Belousov's problem)

M. A. Akivis
Jerusalem Institute of Technology, Jerusalem, Israel
and
V. V. Goldberg
New Jersey Institute of Technology, Newark, NJ, USA

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The authors prove that a local n-quasigroup defined by the equation

$$x_{n+1} = F(x_1, \dots, x_n) = \frac{f_1(x_1) + \dots + f_n(x_n)}{x_1 + \dots + x_n},$$

where $f_i(x_i),\ i, j = 1, \dots, n$, are arbitrary functions, is irreducible if and only if any two functions $f_i(x_i)$ and $f_j(x_j)$, $i \neq j$, are not both linear homogeneous, or these functions are linear homogeneous but $\frac{f_i(x_i)}{x_i} \neq \frac{f_j(x_j)}{x_j}$.

This gives a solution of Belousov's problem to construct examples of irreducible n-quasigroups for any $n \geq 3$.