Finite loops and projective planes

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June 28, 2005

Projective planes are coordinatized by ternary rings (R, T) with an associated "ring" $R = (R, +, \cdot)$, such that $(R, +)$ is a loop with identity 0, and $(R^\#, \cdot)$ is a loop with identity 1, where $R^\# = R - \{0\}$. We consider finite projective planes linearly coordinatized (ie. $T(x, y, z) = xy + z$) by a right distributive (ie. $(x + y)z = xz + yz$) ring R. Most known finite planes are of this sort. We use techniques from finite group theory and loop theory to study such planes and their associated rings and loops. Various partial results suggest that possibly either $(R, +)$ or $(R^\#, \cdot)$ is a group. Much of the analysis involves the following interesting class of loops: Loops $(R, +)$ admitting a group of automorphisms transitive on $R^\#$.