On some subloops of the loop of norm one elements in the split Cayley-Dickson algebra

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Let F be a field. Then by $O(F)$ one denotes the split Cayley-Dickson algebra over F. Recall that $O(F)$ is the set of all "2×2-matrices"

$$\begin{pmatrix} \alpha & a \\ b & \beta \end{pmatrix}$$

such that $\alpha, \beta \in F$ and a, b are vectors in the three-dimensional F-vector space F^3. The multiplication in $O(F)$ is defined by

$$\begin{pmatrix} \alpha & a \\ b & \beta \end{pmatrix} \begin{pmatrix} \gamma & c \\ d & \delta \end{pmatrix} = \begin{pmatrix} \alpha\gamma - a \cdot d & \alpha c + \delta a + b \times d \\ \gamma b + \beta d + a \times c & \beta\delta - b \cdot c \end{pmatrix},$$

where $\cdot$ and $\times$ denote the usual "dot product" and "cross product" in F^3 respectively. If

$$x = \begin{pmatrix} \alpha & a \\ b & \beta \end{pmatrix} \in O(F),$$

then the norm $N(x)$ of x is defined by $N(x) = \alpha\beta + a \cdot b$. Let $\mathrm{SLCD}(F) = \{x \in O(F) : N(x) = 1\}$. Since the equality $N(xy) = N(x)N(y)$ satisfies for any $x, y \in O(F)$, the set $\mathrm{SLCD}(F)$ formes a loop with respect to the multiplication defined in $O(F)$. If $a, b, c \in F^3$, then the mixed product of a, b, c is designated by $[abc]$, that is $[abc] = (a \times b) \cdot c$. Let us denote by $t_{12}(a)$ the element

$$\begin{pmatrix} 1 & a \\ 0 & 1 \end{pmatrix} \in O(F).$$

Theorem 1 *Let F be a field of characteristic $\neq 2$ and k be a subfield of F such that the extension F/k is algebraic. Assume k contains more then three elements. Let a, b, c be linearly independent vectors in F^3. The subloop of* $\mathrm{SLCD}(F)$ *generated by all the elements $t_{12}(ar), t_{12}(b), t_{12}(c)$ where r runs over k, is isomorphic to the loop* $\mathrm{SLCD}(k(\alpha))$ *with $\alpha = [abc]$.*