Abelian inner mappings and nilpotency class greater than two

Piroska Csorgo
Eotvos University, Budapest, Hungary

June 28, 2005

By T. Kepka and M. Niemenmaa if the inner mapping group of a finite loop Q is abelian, then the loop Q is centrally nilpotent. For a long time there was no example of a nilpotency degree greater than two. In the nineties T. Kepka raised the following problem: whether every finite loop with abelian inner mapping group is centrally nilpotent of class at most two. For many years the prevailing opinion has been that all such loops have to be of nilpotency degree two. The converse is always true by Bruck, i.e. the nilpotency class two of the loop Q implies the inner mapping group $I(Q)$ is abelian.

After describing the problem in terms of transversals I tried to characterize by means of group theory the least counterexample. I expected to find enough properties of the counterexample that would refute its existence. By using these results, supposing special properties, I choose some parameters and finally I constructed a conterexample loop Q of order 2^7, such that the multiplication group $M(Q)$ is of order 2^{13}, the inner mapping group $I(Q)$ is elementary abelian of order 2^6, for the normal closure M^0 of $I(Q)$ in $M(Q)$, M^0 is of order 2^{10} and the factor group $M(Q)/M^0$ is elementary abelian of order 2^3, furthermore the nilpotency class of this loop Q is greater than two.