A survey of Osborn loops

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The study of Moufang loops is a time-honored tradition within loop theory, and connects to many different areas of mathematics. The study of CC-loops has taken off in the last few years because of the dissemination of Basarab's theorem (the quotient of a CC-loop by its nucleus is an abelian group). This talk will mostly consist of a mini-course in a common generalization of the two varieties, namely the variety of *Osborn loops*. The picture to have in mind is the following, although I will show a much more elaborate one in the talk.



The definition of Osborn loops is quite natural, and a surprising amount of the basic structure of both Moufang and CC-loops is already present in Osborn loops. I will survey published results (mostly due to Basarab) and some new ones as well. Interestingly, one can even generalize some of the basic results in the theory of commutative Moufang loops to a certain class of Osborn loops. I will also address those in the audience who prefer quasigroups to loops. There is a conjectural relationship between Osborn loops and the (as yet) unstudied class of *conjugacy closed quasigroups*, that is, quasigroups in which the sets of left and right translations are each closed under self-conjugation. If there is time (though I doubt there will be), I will briefly discuss the

one-sided case, namely common generalizations of Bol loops and RCC-loops.