Linear balanced quasigroup identities

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We consider quasigroup identities which contain only the multiplication symbol $\cdot$. Such an identity is *linear* if every variable appears at most once in $s(t)$ and is *balanced* if the set $var(s)$ of variables of s is equal to $var(t)$.

Every linear balanced quasigroup identity is either Belousov (i.e. a consequence of commutativity) or non–Belousov (i.e. implying group isotopy). A. Krapež and M. A. Taylor (Czechoslovak Math. J. 43 (118), (1993)) proved that every set of Belousov identities is equivalent to a single *normal* Belousov identity. Consequently the lattice of nontrivial Belousov varieties is isomorphic to the lattice of odd numbers under divisibility.

Non–Belousov identities are more difficult to handle. W. Förg–Rob and A. Krapež defined *height–preserving* identities ($s = t$ is height–preserving if every variable x is of equal height in s and t) and proved that a quasigroup satisfying such an identity is a T–quasigroup in which $branch(x, s) = branch(x, t)$ for all $x \in var(s)$. Height–preserving identities can be understood as permutations transforming the tree of s to that of t. Belousov identities then correspond to isomorphisms of trees of s and t.

Varieties of quasigroups satisfying height–preserving identities of height up to h can be related to special subgroups of the group S_{2^h}.