On nilpotent Moufang loops

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Despite the importance of Moufang loops in the theory of quasigroups and loops, there are astonishingly few general constructions for nilpotent Moufang loops. The only exceptions are the classes of commutative Moufang loops and Moufang 2-loops (due to Chein's construction and its generalizations). Together with M. Valsecchi, we proved the following.

Theorem 1 *Let M be a group and k a positive integer. Let $f : M \to Z(M)$ and $g : M \times M \to Z(M)$ be maps with the following properties:*

1. *f and g vanish on $f(M) \cup g(M,M)$, that is,*
$$f(f(m)) = f(g(m_1,m_2)) = g(f(m_1),m_2) = g(g(m_1,m_2),m_3) = 1$$
for each $m, m_1, m_2, m_3 \in M$.

2. *g is bilinear and alternating, which means*
$$\begin{aligned} g(m_1m_2,m_3) &= g(m_1,m_3)g(m_2,m_3), \\ g(m_1,m_2) &= g(m_2,m_1)^{-1}, \\ g(m,m) &= 1 \end{aligned}$$
for each $m, m_1, m_2, m_3 \in M$.

3. *f satisfies*
$$f(m_1m_2) = f(m_1)f(m_2)g(m_1,m_2)^3 \tag{1}$$
for all $m_1, m_2 \in M$ and $f(m)^k = 1$ for all $m \in M$.

Define the operation
$$(i_1,m_1) \cdot (i_2,m_2) = (i_1+i_2, m_1m_2f(m_1)^{i_2}g(m_1,m_2)^{i_1+2i_2}) \tag{2}$$
on the set $L = \mathbf{Z}_k \times M$. Then, $(L,\cdot)$ is a Moufang loop with unit $(0,1)$.

It turns out that these loops can be very effectively used in the following areas:

1) Classification of Moufang loops of order p^5, $p > 3$.

2) Structural properties and examples of Moufang loops with central associators.

3) Minimally nonassociative Moufang p-loops.