Powers and Alternative Laws

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A groupoid is alternative if it satisfies the alternative laws: $x(xy) = (xx)y$ and $x(yy) = (xy)y$. Let A be the free alternative groupoid with generator x. An nth power of x is an element of A which is obtained by multiplying x with itself n times. Of course, nth powers of x need not be unique since there are many ways to parenthesize the multiplication. Each application of an alternative law passes from one nth power of x to another and a basic question is - for which values of n are nth powers of x unique? We show that the answer is only when $n \leq 5$.

The technique used is to study the action of multiplication by 2 modulo n. In dynamical terms, this is a quotient of the original action of application of an alternative law (or the inverse of such an application), but is much simpler. In particular, if we show that this map has orbits which are not complete, we show that powers cannot be unique. We also use this action to say something about the problem of finding alternative loops without two-sided inverses.